PHIL170 - DISCUSSION January 29, 2010

LOGIC: Is the study of valid forms of logical reasoning.

2 Types of Arguments

Inductive: Premeses only provide some degree of support for the conclusion. There is a degree of uncertainty. (use words like likely / Probably

Deductive: Premises fully supports conclusion.

2 Properties of Arguments

Validity:

Soundness: whether the argument is true or not

EXAMPLES: Antecedent False consequent False

P1) If Bush is President, then laura is the first lady.

P2) Bush is not the president ← True

C) Therefore Laura is not the first lady. ← True

For ($P_1$), this is an if P, then Q sentence.

- This premis is "Denying the antecedent"

• This argument is intuitively invalid because Laura could be the first lady even if Bush is not President.

- she could have divorced Bush and married Obama.

| P | Q | P∪Q |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |

Invalid, Deductive argument example: 2.1.h from HW-1:

7 = not
V = Or

If 7D V 7E, then G
∴ If 7D, Then G

If i dont diet or dont exercise, ill gain weight
if I dont diet, ill gain weight

## Validity

February 1, 2010

Validity is about preserving truth.

valid argument: The truth of the premises quarentee the truth of the conclusion.

Ex.: If you oppose the healthcare bill, then you value a freemarkert society
You dont oppose the healthcare bill.
So you must not value a freemarket society.

unterex.: If DC's in Maryland, DC's not in the U.S.
DC is not in Maryland
So DC is not in the U.S.

Ex. 2:

February 1, 2010

Lecture 3 - Structure & Form of Sentential Logic

Sentencial Logic: Logic dealing with sentences.

- The most basic unit is always a complete sentence

Simple vs. compound sentences

simple sentences:

- "Scuba steve swims"
- "Becky prefers doughnuts with sprinkles to those with jelly"

compound sentences:

- "Lance & Giovani ride their bikes to school." *and*
- "Politicians don't like being indicted". *dont*
  (Politician's like being indicted)
- "If lady Gaga isnt crazy, then I'm a walrus" *If, then*

sentential operator: An expression used to build up compour sentences out of simple ones.

- Example: In "Polititions dont like being indicted" the sentential operator is "don't". ~~This word makes~~
  ~~For~~ - This negation allows for 2 sentences:
  1 Politicians like being indicted
  2 Politicians dont like being indicted.

More compound sentences:

A declarative sentence is ~~composed~~ compound if it contains another complete declarative sentence as a component.

Examples: "Peter likes peanut butter and Jason likes Jam"

"Darcy thought that Jane was in love with Hugh."

- Anything with a "that" clause.

Distinguishing simple from compound

"John doubts that the spider is poisonous."

"John doubts that sunlight is dangerous"

- The declarative component sentence can be replaced ∴ compound
- Negated sentences are compound.

sentential operaters: ~~An expression containing "blanks"~~

① ... and ...
② ... or ...
③ If ... then ...
④ It is not the case that...
⑤ ... if and only if ...

## Symbols for ~~sentence~~ sentential operators

"... and ..." : $\underline{P} \bullet \underline{Q}$ (conjunction)

"... or ..." : $\underline{P} \vee \underline{Q}$ (disjunction)

f... then ..." : $\underline{P} \supset \underline{Q}$ (conditional) P is antecedent ; Q is consequen

..if and only if..." : $\underline{P} \equiv \underline{Q}$ (biconditional)

"not" : $\underline{P} \sim \underline{Q}$ (negation)

LECTURE 4 - TRANSLATING ENGLISH TO SYMBOLS February 3, 2010

## Symbolizing Simple Sentences

Simple English sentences are replaced with single capital letters to become simple formulas.
- Ex. S = Scuba Steve Swims
- Ex. A = Avalanches are dangerous.
- Ex. B = Becky prefers doughnuts with sprinkles to the ones with jelly inside.

Must use different sentence letters for different simple sentences in th same compound sentence or argument.
- But if the same simple sentence occurs more than once in the same argument, use the same sentence letter.

## Symbolizing Compound Sentences

Compound sentences are translated by symbolizing the simple component and adding operator symbols and parentheses as needed:
- ex. Lance & Giovanni ride their bikes to school $(L \cdot G)$

Lance rides his bike to school; Giovanni rides his bike to school)
- ex. Politicians don't like being indicted $\sim P$
  (Politicians like being indicted)
- ex. If lady Gaga isn't crazy, Im a walrus $(\sim L \supset W)$
  (Lady Gaga isn't crazy; I'm a Walrus)

## Negation

- Ex. It is not the case that either Tom is hungry or Jerry is tired. $\sim (T \vee J)$
- Ex. Its not the case that Tom is hungry if and only if Jerry is not tired $\sim (T \equiv \sim J)$

## Major Operator

The operator that determines whether the overall formula is a conditional, biconditional, conjunction, etc.

### Major Operator in Symbols:

x. $((A \cdot G) \boxed{\supset} R)$ The horseshoe is the major operator, signifying that this sentence is a conditional.

x. $(R \cdot (F \supset S))$ The dot is the major operator, signifying that this sentence is a conjunction

## Major Operator in English

Ex. 1 It is $\boxed{\text{not}}$ true that Hartounian is tall and strong [Negation]

Ex. 2 If Ben and Silla wait their turn, everyone will be relieved. [conditional]

Ex. 3 $\boxed{\text{Either}}$ both Jacques and Ludmilla will be elected $\boxed{\text{or}}$ neither will [disjunction]

## Steps for symbolizing

1. Identify simple components... mentally replace with letters.
2. Identify major operator in English and replace with appropriate symbol
3. Repeat #2 with subformula, adding parenthases as necessary.

Ex. Either you'll go to jail and you'll pay a fine or you'll do community service

- $\boxed{\text{Either}}$ J and F $\boxed{\text{or}}$ C.
- ((J $\boxed{\text{and}}$ F) v C)
- $((J \cdot F) \vee C)$

## Step-by-step example

If Chile will join the pact if and only if Denmark joins, then unless Spain objects if and only if Portugal complains, Columbia will protest.

* "unless" is used as "or"

- $\boxed{\text{If}}$ C iff D, $\boxed{\text{Then}}$ S iff P, or T
- $(C \text{ iff } D) \supset (S \text{ iff } P \text{ or } T)$
- $(C \equiv D) \supset ((S \text{ iff } P) \text{ or } T)$
- $(C \equiv D) \supset ((S \equiv P) \vee T)$

## UNIT 4

| | | | |
|---|---|---|---|
| If P, then Q | $(P \supset Q)$ | Q only if P | $(Q \supset P)$ or $(\sim P \supset \sim Q)$ |
| if P, Q | $(P \supset Q)$ | P only if Q | $(P \supset Q)$ or $(\sim Q \supset \sim P)$ |
| Q if P | $(P \supset Q)$ | P unless Q | $(\sim Q \supset P)$ or $(P \vee Q)$ |
| Q provided P | $(P \supset Q)$ | not P unless Q | $(\sim Q \supset \sim P)$ or $(\sim P \vee Q)$ |
| provided P, then Q | $(P \supset Q)$ | | |

EXAM ON MONDAY, MARCH 1st

- PRINT OUT TRANSLATION SHEET ON BLACKBOARD.
- LOOK UP TRUTH TABLES ON THIS LECTURE ONLINE.

## THE DAY AFTER SNOWMAGGEDEN - LECTURE 7

February 15, 2010

### "if" versus "only if"

"if" or another phrase that means "if" ("provided that") means that the antecedent is coming.

"only if" or "only provided" means that the consequent is coming.

Ex. 1 - "can I get a ride on your scooter"

2 - "only if you give me five bucks"

1 - " here's five bucks

2 - (Guy ~~two~~ two drives off)

whats happening:

ride → $R \supset F$ ← five bucks

- He gets 5 bucks but then doesnt have to give a ride.
- It would be different if he said "if you give me five bucks"
- because $F \supset R$, This is if ~~R~~F, then R.

### "not both" and "neither...nor..."

- "not both" is a negated conjunction It's a "not" in front of ~~both~~ "both P and Q" $\sim(P \cdot Q)$
- "neither... nor..." is a negated disjunction Its a "not" in front of "p or q" $\sim(P \vee Q)$

Ex: "Only provided that neither John nor Mary go to the party will either Steve or Rebecca (but not both) attend."

- only provided that neither J nor M, S or R but not both S and R
- S or R but not both S and R $\supset$ neither J nor M

[S or R • not both S and R] $\supset \sim(J \vee M)$

same { $[(S \vee R) \cdot \sim(S \cdot R)] \supset \sim(J \vee M)$, or
$[(S \vee R) \cdot (\sim S \cdot \sim R)] \supset (\sim J \vee \sim M)$

### Because

Because cannot be translated in logic, so "because" sentences are simple sentences.

Ex. "The roof fell down because of all the snow"

answer: S

This ~~is~~ a complex sentence, but there is no operator for "because"

Invalid, True premeses, false conclusion

Pg. 85 This is what we want them to be

February 22, 2010

| p | q | r | $\overset{T}{(p \cdot (q \cdot r))}$ | $\overset{T}{p \supset (r \vee \sim q)}$ ~~$p \supset q \cdot (r \vee \sim q)$~~ | $\overset{F}{p \supset (\sim q \vee \sim r)}$ |
|---|---|---|---|---|---|
| T | T | T | | | T F F F |
| T | T | F | | | T |
| T | F | T | | | T |
| T | F | F | | | T |
| F | T | T | | | T |
| F | T | F | | | T |
| F | F | T | | | T |
| F | F | F | | | T |

only one possible row for a counter example so we dont have to worry about the othe rows.

when antecedent is false, conditional is true

This is invalid, because a counter example is possible.

Ex. 2: 5.2.a

| | | | $\sim(r \vee q) \overset{T}{\supset} \sim(p \vee r)$ | $\sim(p \vee q) \overset{T}{\supset} \sim(q \cdot r)$ | $(p \vee \sim q) \overset{T}{\supset} \sim(r \cdot p)$ | $\overset{F}{p \supset (\sim r \vee \sim q)}$ |
|---|---|---|---|---|---|---|
| T | T | T | F T T F T | F T | T F F | F |
| T | T | F | | | | T |
| T | F | T | | | | |
| T | F | F | | | | |
| F | T | T | | | | |
| F | T | F | | | | |
| F | F | T | | | | |
| F | F | F | | | | |

Valid, no counterexample

## SHORT TRUTH TABLES

p q r s t w $((p \vee q) \supset \sim p)$ | $((r \cdot s) \vee t)$ | $(t \supset (w \supset s))$ || $(p \supset s)$ X

T F F T T T T T F | F F F T T | T T F T F || T F F

F

F

Invalid, counter example.

- start with the conclusion, because the conclusion is a conditional and to make a conditional false is to have a true antecedent and false consequent.

p q r s t $((p \vee q) \supset r)$ | $((r \vee s) \supset \sim t)$ || $(p \supset \sim t)$

T T T T T T T | T T F F T || T F F T

Valid, no conterexample

## ON EXAM

Definitions from units 1, 5, 6

- Validity, come up with a counterexample if invalid.
- Identify simple vs. compound + the simple parts.
- Identify major operators. (in english + symbols)
- Translate sentences from english to symbols.
- Truth tables for the 5 operators.
- Determine the truth values of complex formulas (w/unknowns)
- long truth table method for validity.
- Short truth table method.
- Use truth tables to determine:

  whether a statement form is tautology, controdiction, or contingency,

  whether a statement logically implies or is logically equivelant to another statement.

# Lecture 10

2/24/10

OTHER THINGS WE CAN DO WITH TRUTH TABLES:

TAUTOLOGY:

- Are always true
- A statement form thats true for any sentence you plug in due to the consistancy of the operators.

Example: $P \vee \sim P$ , always true

CONTRODICTION:

- Are alway false
- Any sentences you put in, the statement will always be false

Example: $P \bullet \sim P$

CONTINGENCY:

- True for some substitutions, false for others.

$P \supset (q \bullet r)$

LOGICAL IMPLICATION:

LOGICAL EQUIVELANCE:

- IF ONE [illegible] IS FALSE, THE [illegible]
- IF ONE CONJUNCT [illegible]
- IF THE ANTECEDENT IS FALSE [illegible]
- IF THE CONSEQUENT IS TRUE, THE COND[ITIONAL] IS TRUE

## TRUTH VALUES FOR SENTENTIAL CONNECTIVES

### CONJUNCTION

| p | q | (p · q) |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | F |

- Conjunctions are true only when both conjuncts are true

### DISJUNCTIONS

| p | q | (p ∨ q) |
|---|---|---|
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | F |

- Disjunctions are false only when both disjuncts are false

### BICONDITIONALS

| p | q | (p ≡ q) |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | T |

- Biconditionals are true only when the truth values of the components are the same

### NEGATIONS

| p | ~p |
|---|---|
| T | F |
| F | T |

4 negation of p has the opposite truth value of p

### CONDITIONAL

| p | q | (p ⊃ q) |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |

- Conditionals are false only when the antecedent is true and the consequent is false

# THE PROOF METHOD

March 3, 2010

## Constants vs. variables

- statement constant: Letter used to abbreviate a sentence.
- statement variable: letter used as a substitution for any statement.

  Ex. p, q, r, ...
- statement: $A \supset (B \cdot C)$
- Statement form: $p \supset (q \cdot r)$ (form of the $A \supset (B \cdot C)$)
  - $A \supset (B \cdot C)$ is a substitution instance of $p \supset (q \cdot r)$
- substitution instance: A statement thrown in for variables $p \supset (q \cdot r)$

  *Must substitute the same statement for repeated occurences of the same variable in a form.

  Ex.: statement form: $(p \cdot q) \supset r$

  $(A \cdot G) \supset T$

  $(A \cdot (B \vee C)) \supset (A \vee C)$

  All of the same form, because they are all conditionals. w/conjunction as antecedent

  - look at the major operator.
- There are an <u>infinite</u> number of substitutions for each form.

~~Substitution Instances and Major operators~~

## Substitution Rules

Can't substitute different statements for the same variable.

Can substitute the same statement for different variables.

$p \supset (q \supset p)$ { Ex. $A \supset (\sim A \supset A)$ Good!

Ex. $H \supset (H \supset G)$ wrong! different variables for p!

$A \supset (A \supset A)$ Good!

## RULES FOR CONJUNCTION: ① SIMPLIFICATION (SIMP)

- For when conjunction is major operator: ...

1. From a conjunction as a premise, we can infer either of the conjuncts seperately as a conclusion.

   Ex. $\dfrac{P \cdot q}{/\therefore P}$ $\dfrac{P \cdot q}{/\therefore q}$

2. CONJUNCTION (CONJ): Given any 2 statements, we can infer their conjunction.

   Ex. $P$

   $\underline{q}$

## RULES FOR DISJUNCTION

1. **DISJUNCTIVE SYLLOGISM (DS):** Given a disjunction and the negation of one of the disjuncts, you may infer the other disjunct.

Ex.
$$\begin{array}{l} p \vee q \\ \underline{\sim p} \\ /\therefore q \end{array} \qquad \begin{array}{l} p \vee q \\ \underline{\sim q} \\ /\therefore p \end{array}$$

2. **ADDITION (ADD):** Given any statement, you may infer any disjunction that includes that statement as one of the disjuncts.

Ex.
$$\begin{array}{l} \underline{p} \\ /\therefore p \vee q \end{array} \qquad \begin{array}{l} \underline{q} \\ /\therefore p \vee q \end{array}$$

## RULES FOR CONDITIONAL

1. **MODUS PONENS (MP):** Given a conditional and the antecedent of that conditional, you are permitted to infer the consequent of the conditiona

Ex.
$$\begin{array}{l} p \supset q \\ \underline{p} \\ /\therefore q \end{array}$$

2. **MODUS TOLLENS (MT):** Given a conditional and the negation of the consequent, of the conditional, you can infer the negation of the antecedent of the conditional

$$\begin{array}{l} p \supset q \\ \underline{\sim q} \\ /\therefore \sim p \end{array}$$

3. **HYPOTHETICAL SYLLOGISM (HS):** Given 2 conditionals in which the conseque of the 1st is the antecedent of the second, You may infer a conditiona whose antecedent is the antecedent of the 1st and whose consequent is the consequent of the 2nd.

ex.
$$\begin{array}{l} p \supset q \\ \underline{q \supset r} \\ /\therefore p \supset r \end{array}$$

4. **DILEMMA (DIL):** Given 2 conditionals and the disjunction of their antecedents, we may infer the disjunction of their consequents.

ex.
$$\begin{array}{l} p \supset q \\ r \supset s \\ \underline{p \vee r} \end{array}$$

## LECTURE 12 - THE PROOF METHOD

March 8, 2010

### CONSTRUCTING PROOFS

- Start w/ premises
- Proceed to derive conclusion

### JUSTIFICATION

- Each step you make in a proof must be justified
  - Justified if a premise.
  - Justified if follows a previous step using a rule of inference.

### HOW TO WRITE JUSTIFICATIONS

- written to the right of each step.
  - If the step's a premise, write "Pr"
  - If not a premise:
    1. write the rule used to derive it.
    2. write the previous step(s) that served as premise(s) in the applicat of the rule.

EXAMPLE:

| | | |
|---|---|---|
| 1. | $(A \vee B) \cdot \sim F$ | Pr. |
| 2. | $G \supset F$ | Pr. |
| 3. | $H \supset G$ | Pr. /∴ $(\sim G \cdot \sim H) \cdot (A \vee B)$ |
| 4. | $\sim F$ | Simp 1 |
| 5. | $(A \vee B)$ | Simp 1 |
| 6. | $\sim G$ | MT 2,4 |
| 7. | $\sim H$ | MT 3,6 |
| 8. | $\sim G \cdot \sim H$ | Conj. 6,7 |
| 9. | $(\sim G \cdot \sim H) \cdot (A \vee B)$ | Conj 5,8 |

### PROOFS VS. DERIVATIONS

- Derivation - Any sequence of justified steps.
- we derive statements from other statements using our rules.
- A derivation that ends with a particular (premeditated) conclusion is a proof.

### TWO BASIC STRATEGIES FOR CONSTRUCTING PROOFS:

1) "TOP DOWN"
   - Dont look at conclusion, just begin to derive everything I can, then look at the conclusion after a while.
2) "Bottom UP" - start by looking at conclusion, do steps backward seeing wha you will need to do to get to the conclusion.

## THINKING AHEAD A LITTLE...

| | | |
|---|---|---|
| 1. | $(A \vee B) \supset \sim C$ | Pr. |
| 2. | $C \vee D$ | Pr. |
| 3. | $A$ | Pr. /∴ D |

we want to us DS on line 2 to get ~C.
So we need to use MP on line 1 to get ~C.
So we need to use addition on line 3.

| | | |
|---|---|---|
| 4. | $A \vee B$ | Add 3 |
| 5. | $\sim C$ | MP 1,4 |
| 6. | $D$ | DS 2 |

## THINGS TO KEEP IN MIND

- There will be more than 1 way to do a proof. - from using the same rules in a different order; to just using different rules.
- You can use premises more than once.
  - You may not need to use all of the premises at all.
  - Premises can be listed in any order and you can use them in any orde
- It's okay to have extra steps. As long as everything done is justified, then nothing wrong with going in a fruitless direction first.

## TRICKY LITTLE PROOF

Example:

| | | |
|---|---|---|
| 1. | $A \vee B$ | Pr. |
| 2. | $B \supset C$ | Pr. |
| 3. | $\sim A \cdot \sim C$ | Pr. /∴ $F \supset G$ |

How can $F \supset G$ be concluded when neither F nor G are in the premises.

| | | |
|---|---|---|
| 4. | $\sim A$ | Simp 3 |
| 5. | $B$ | DS 4,1 |
| 6. | $\sim C$ | Simp 3 |
| 7. | $\sim B$ | MT 2,6 |
| 8. | $B \vee (F \supset G)$ | Add 5 |
| 9. | $F \supset G$ | DS 7,8 |

contradiction

- This contradiction can be used to our advantage here to derive our conclusion.

8.e. PROOF EXAMPLES

| | | |
|---|---|---|
| 1. | $F \supset (G \cdot \sim H)$ | Pr. |
| 2. | $Z \supset H$ | Pr. |
| 3. | $F$ | Pr. /∴ $\sim Z$ |
| 4. | $G \cdot \sim H$ | MP 1,3 |
| 5. | $\sim H$ | Simp 4 |
| 6. | $\sim Z$ | MT 2,5 |

8.L.

| | | |
|---|---|---|
| 1. | $(A \vee B) \supset (C \vee D)$ | Pr. |
| 2. | $C \supset E$ | Pr. |
| 3. | $A \cdot \sim E$ | Pr. /∴ $D \vee W$ |
| 4. | $A$ | Simp 3 |
| 5. | $\sim E$ | Simp 3 |
| 6. | $\sim C$ | MT 2,5 |
| 7. | $A \vee B$ | Add 4 |
| 8. | $C \vee D$ | MP 1,7 |
| 9. | $D$ | DS 6,8 |
| 10. | $D \vee W$ | Add 9 |

D Add
$A \vee B$ MP
$\sim C$ DS

what's $\sim\sim A$?

Read replacement substitution Rules in Book

① <u>DeMorgans</u> (DeM)

$$\frac{\sim(P \vee Q)}{\sim P \cdot \sim Q} \qquad \frac{\sim(P \cdot Q)}{\sim P \vee \sim Q}$$

② <u>Double Negation</u> (DN)

$$\frac{\sim\sim P}{P} \qquad \frac{P}{\sim\sim P}$$

③ <u>Exportation</u> (Exp)

$\cancel{(P \supset Q)}$

$$\frac{P \supset (Q \supset R)}{(P \cdot Q) \supset R}$$

# Proof practice

03/22/10

) Tricks with replacement rules.

Have: $\sim A$

want: $\sim(A \cdot B)$

1. $\sim A$ — Pr. /∴ $\sim(A \cdot B)$
2. $\sim A \vee \sim B$ — Add 1
3. $\sim(A \cdot B)$ — DEM 2

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) have: $p \equiv q$, $\sim p$

want: $\sim q$

| | | | |
|---|---|---|---|
| 1. $p \equiv q$ | Pr. | $p \equiv q$ | |
| 2. $\sim p$ | Pr. /∴ $\sim q$ | $p$ | |
| 3. $(p \supset q) \cdot (q \supset p)$ | BE 1 | | |
| 4. $q \supset p$ | Simp 3 | $p \supset q$ | |
| 5. $\sim q$ | MT 2,4 | $q$ | MP |

---

)

Have: $q$

want: $p \supset q$

1. $q$ — Pr. /∴ $p \supset q$
2. $\sim p \vee q$ — Add 1
3. $p \supset q$ — CE 2

---

1)

1. $\sim p$ — Pr. /∴ $p \supset q$
2. $\sim p \vee q$ — Add 1
3. $p \supset q$ — CE 2

5) 1. $\sim(P \vee q)$ Pr. /∴ $\sim P$
2. $\sim P \cdot \sim q$ DeM 1
3. $\sim P$ Simp 2

---

6) 1. $P$ Pr.
2. $q$ Pr. /∴ $P \equiv q$ ∴? $(P \supset q) \cdot (q \supset P)$
3. $\sim q \vee P$ Add 1
4. $q \supset P$ CE 3
5. $\sim P \vee q$ Add 2
6. $P \supset q$ CE 5
7. $(P \supset q) \cdot (q \supset P)$ Conj 6
8. $P \equiv q$ BE 7

---

7) 1. $\sim(P \equiv q)$ Pr.
2. $\sim[(P \supset q) \cdot (q \supset P)]$ BE 1
3. $\sim(P \supset q) \vee \sim(q \supset P)$ DEM 2
4. $\sim(\sim P \vee q) \vee \sim(\sim q \vee P)$ CE (twice)
5. $(\sim\sim P \cdot \sim q) \vee (\sim\sim q \cdot \sim P)$ DEM (TWICE)
~~6. $(P \cdot \sim q) \vee (q \cdot \sim P)$ DN (Double Negation)~~

... Dont worry about this one for now. –

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8) 1. $\sim(P \supset q)$ Pr. /∴ $P$
2. $\sim(\sim P \vee q)$ CE 1
3. $\sim\sim P \cdot \sim q$ DEM 2
4. $P \cdot \sim q$ DN 3 (Double Negation)
5. $P$ Simp ~~3~~ 4
6. $\sim q$ Simp 4

EXAM 2. Wednesday April $7^{th}$
Final May $19^{th}$ 8AM
Homework prob. 7.9.g pg. 134-135 in book

March 24, 2010

* Proofs show us that the conclusion follows validly from the premises. ∴ The premises and conclusion constitute a valid argument.

MORE PROOF EXAMPLES

| | | | |
|---|---|---|---|
| 7.9.i | 1. | $(\sim P \vee \sim Q) \supset (\sim R \vee \sim S)$ | Pr. |
| | 2. | $P \supset T$ | Pr. |
| | 3. | $\sim W \supset (\sim T \cdot \sim Z)$ | Pr. |
| | 4. | $(\sim S \supset Z) \cdot \sim (X \cdot Y)$ | Pr. |
| | 5. | $\sim W \vee (X \cdot Y)$ | Pr. /∴ $\sim R \cdot \sim W$ |
| | 6. | $\sim S \supset Z$ | SIMP 4 |
| | 7. | $\sim (X \cdot Y)$ | SIMP 4 |
| | 8. | $\sim W$ | DS 5,7 |
| | 9. | $\sim T \cdot \sim Z$ | MP 3,8 |
| | 10. | $\sim T$ | SIMP 9 |
| | 11. | $\sim P$ | MT 2,10 |
| | 12. | $\sim P \vee \sim Q$ | ADD 11 |
| | 13. | $\sim R \vee \sim S$ | MP 1,12 |
| | 14. | $\sim Z$ | SIMP 9 |
| | 15. | $\sim \sim S$ | MT 6,14 |
| | 16. | $\sim R$ | DS 13,15 |
| | 17. | $\sim R \cdot \sim W$ | CONJ 8,16 |

| | | | |
|---|---|---|---|
| 3.5.n | 1. | $\sim ((A \vee B) \vee (C \vee D))$ | Pr. /∴ $\sim D$ |
| | 2. | $\sim (A \vee B) \cdot \sim (C \vee D)$ | DEM 1 |
| | 3. | $\sim (A \vee B) \cdot (\sim C \cdot \sim D)$ | DeM 2 |
| | 4. | $\sim C \cdot \sim D$ | SIMP 3 |
| | 5. | $\sim D$ | SIMP 4 |

8.5.O

| | | |
|---|---|---|
| 1. | $B \supset (C \supset E)$ | Pr. |
| 2. | $E \supset \sim(J \vee H)$ | Pr. |
| 3. | $\sim S$ | Pr. |
| 4. | $J \vee S$ | Pr. / $\therefore B \supset \sim C$ |
| 5. | $(B \cdot C) \supset E$ | Exp. 1 |
| 6. | $J$ | DS 3,4 |
| 7. | $J \vee H$ | ADD 6 |
| 8. | $\sim\sim(J \vee H)$ | DN 7 |
| 9. | $\sim E$ | MT 2,8 |
| 10. | $\sim(B \cdot C)$ | MT 5,9 |
| 11. | $\sim B \vee \sim C$ | DeM 10 |
| 12. | $B \supset \sim C$ | CE 11 |

8.5.0

| | | |
|---|---|---|
| | | $\sim((A \supset B) \cdot (B \supset A))$ |
| 1. | $A$ | Pr. |
| 2. | $\sim B$ | Pr. /∴ $\sim(A \equiv B)$ |
| 3. | $A \cdot \sim B$ | conj 1,2 |
| 4. | $\sim B \cdot A$ | conj 1,2 |
| 5. | $\sim B \vee A$ | Add 2 |
| 6. | $B \supset A$ | CE 5 |
| 7. | $\sim B \supset \sim A$ | contrap 6 |
| 8. | $\sim A$ | MP 2,7 |
| 9. | $\sim A \vee B$ | Add 8 |
| 10. | $A \supset B$ | CE 9 |
| 11. | $((A \supset B) \cdot (B \supset A))$ | conj 6,10 |
| 12. | $A \equiv B$ | BE 11 |
| ~~13~~ | $\cancel{(\sim B \supset \sim A)}$ | ~~contrap~~ |

1.6.0

| | | |
|---|---|---|
| 1. | $B \supset (C \supset E)$ | Pr. |
| 2. | $E \supset \sim(J \vee H)$ | Pr. |
| 3. | $\sim S$ | Pr. |
| 4. | $J \vee S$ | Pr. /∴ $B \supset \sim C$ |
| 5. | $(B \cdot C) \supset E$ | EXP. 1 |
| 6. | $J$ | DS 4 |
| 7. | $E \supset (\sim J \cdot \sim H)$ | DeM. 2 |
| 8. | $\sim(\sim J \cdot \sim H) \supset \sim E$ | contrap 7 |
| 9. | $J \cdot H \supset \sim E$ | DN 8 |
| 10. | $J \supset (H \supset \sim E)$ | EXP 9 |
| 11. | $H \supset \sim E$ | MP 6,10 |

3.7.n

| | | |
|---|---|---|
| 1. | $(P \cdot S) \supset (T \vee W)$ | Pr. |
| 2. | $\sim T \equiv \sim(M \cdot O)$ | Pr. |
| 3. | $\sim(W \vee (\sim S \vee M))$ | Pr. $\sim P$ |
| 4. | $\sim A \supset P$ | Pr. $/ \therefore A$ |
| 5. | $P \supset (S \supset (T \vee W))$ | Exp. 1 |
| 6. | $\sim P \supset \sim\sim A$ | contrap 4 |
| 7. | $\sim P \supset A$ | DN 6 |
| ~~8.~~ | $\cancel{(\sim T \cdot (\sim(M \cdot O)))}$ | |
| 8. | $(\sim T \supset \sim(M \cdot O)) \cdot (\sim(M \cdot O) \supset \sim T)$ | BE 2 |

3.7.n

| | | |
|---|---|---|
| 1. | $G \supset (H \cdot I)$ | Pr. |
| 2. | $J \supset (H \cdot K)$ | Pr. |
| 3. | $((L \supset \sim G) \cdot M) \supset N$ | Pr. |
| 4. | $(M \supset N) \supset (L \cdot J)$ | Pr. $/ \therefore I \vee K$ |
| 5. | $(L \supset \sim G) \supset (M \supset N)$ | EXP. 3 |
| 6. | $\sim G \vee (H \cdot I)$ | CE 1 |
| 7. | $\sim J \vee (H \cdot K)$ | CE 2 |
| 8. | $\sim(H \cdot I) \supset \sim G$ | contrap 1 |
| 9. | $\sim(H \cdot K) \supset \sim J$ | contrap 2 |

Conditional Proof (C.P.) March 29, 2010

Ex. 1)

| | | |
|---|---|---|
| | 1. $(A \vee B) \supset (C \cdot D)$ | Pr. $A \supset C$ |
| assumption | 2. $A$ | ASSP (C.P) |
| indent | 3. $A \vee B$ | Add 2 |
| | 4. $C \cdot D$ | MP 1,3 |
| | 5. $C$ | Simp 4 |
| | 6. $A \supset C$ | CP 2-5 |

1) write the derived consequent below the line
2) write C.P. (the justifying rule)
3) write the line #'s betw the assumption and the conditional.

we use C.P.'s when we're trying to prove a conditional.

Ex 2)

| | |
|---|---|
| 1. $(A \vee B) \supset \sim (C \vee D)$ | Pr. |
| 2. $(\sim C \cdot E) \supset (F \cdot \sim O)$ | Pr. |
| 3. $(F \vee H) \supset (J \cdot \sim K)$ | Pr. /∴ $(A \cdot E) \supset \sim K$ |
| 4. $A \cdot E$ | ASSP. (C.P.) |
| 5. $A$ | Simp. 4 |
| 6. $E$ | Simp. 4 |
| 7. $A \vee B$ | Add. 5 |
| 8. $\sim (C \vee D)$ | MP 1,7 |
| 9. $\sim C \cdot \sim D$ | DeM 8 |
| 10. $\sim C$ | Simp. 9 |
| 11. $\sim C \cdot E$ | conj. 6,10 |
| 12. $F \cdot \sim O$ | MP 2,11 |
| 13. $F$ | Simp. 12 |
| 14. $F \vee H$ | Add 13 |
| 15. $J \cdot \sim K$ | MP 3,14 |
| 16. $\sim K$ | Simp 15 |
| 17. $(A \cdot E) \supset \sim K$ | C.P. 4-16 |

- Lines 4–16 are reffered to as a subproof.
- The line/arrow is called the "scope marker".
- once the consequent is derived, the lines within the C.P. are "off limits"!