

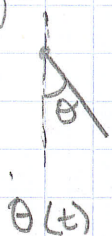
1. $y' = ry$ $y = f(t)$

Applications: Rate of change of bacteria in medium is directly proportional to the amount present
 Radioactive decay
 Continuous compounding

2. $m \frac{dy}{dt} = ma = F = -mg$ (Falling body without air resistance)

With resistance: $m \frac{dy}{dt} = -mg + \gamma v$, $\gamma > 0$

3. Rigid Pendulum



$$\frac{d^2\theta}{dt^2} + \frac{g}{L} \sin\theta = 0$$

Note: The "order" of a diff. equation refers to the highest order of the derivatives that appear.
 • Diff. equations of 1 unknown variable are 'ordinary'
 • Above are explicit diff. equations - $\frac{dy}{dt}$ appears by itself

First Order Ordinary Differential Equations (explicit)

$$y' = f(t, y)$$

1. $y = Ke^{rt} \Rightarrow y' = re^{rt} = ry$
 K any constant

Linear Equations with constant coefficients:

$y = f(t)$
 $y' = ay + b$ a, b constants

$$\frac{dy}{dt} = ay + b$$

$a = 0; \frac{dy}{dt} = b \rightarrow y = bt + c$

$a \neq 0; \frac{dy}{dt} = ay + b$

$$\int \frac{dy}{dt} \cdot \frac{dt}{ay+b} = \int 1 dt = t + c$$

$$\frac{1}{a} \ln|ay+b| = t + c \rightarrow \ln|ay+b| = a(t+c)$$

$$ay+b = \pm e^{\frac{ac}{a}} e^{at}$$

$$|ay+b| = e^{\frac{ac}{a}} e^{at}$$

$$ay = Ke^{at} - b \rightarrow y = \frac{1}{a} (Ke^{at} - b)$$

