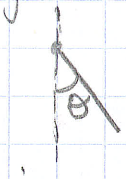
http://www.math.umd.edu/~rll/courses/math246.f10

1. $y' = ry$ $\quad y = g(t)$ Applications: Rate of change of bacteria in medium is directly proportional to the amount present
Radioactive decay
Continuous compounding

2. $m\frac{dv}{dt} = ma = F = -mg$ (Falling body without air resistance)

With resistance: $m\frac{dv}{dt} = -mg + \gamma v \quad , \gamma > 0$

3. Rigid Pendulum



$$\frac{d^2\theta}{dt^2} + \frac{g}{L}\sin\theta = 0$$

$\theta(t)$

Note: The "order" of a diff. equation refers to the highest order of the derivatives that appear.
- Diff. equations of (t) 1 unknown variable are "ordinary"
- Above are explicit diff. equations – $\frac{dy}{dt}$ appears by itself

First Order Ordinary Differential Equations (explicit)

$y' = f(t, y)$

1. $y = Ke^{rt} \Rightarrow y' = re^{rt} = ry$
   K any constant

Linear Equations with constant coefficients:

$y = g(t)$

$y' = ay + b \quad$ a, b constants

$\frac{dy}{dt} = ay + b$

$a = 0;\ \frac{dy}{dt} = b \rightarrow y = bt + C$

$a \neq 0;\ \frac{dy}{dt} = ay + b$

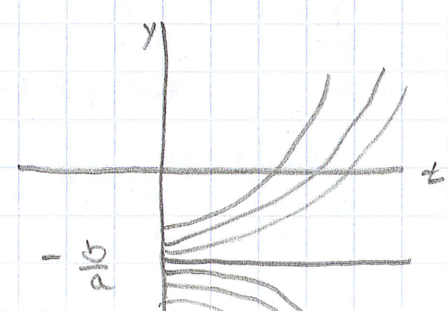
$\int \frac{dy}{dt} \cdot \frac{dt}{ay+b} = \int 1\, dt = t + C$

$\frac{1}{a}\ln|ay+b| = t + C \longrightarrow \ln|ay+b| = a(t+c)$

$|ay+b| = e^{ac}e^{at}$

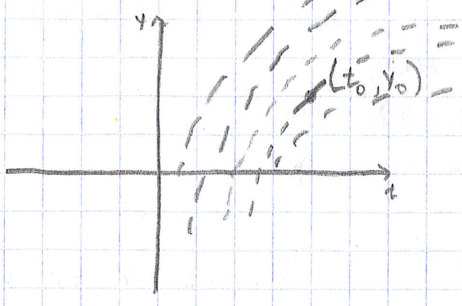
$ay + b = \underbrace{\pm e^{ac}}_{\rightarrow K} e^{at}$

$ay = Ke^{at} - b \longrightarrow \boxed{y = \frac{1}{a}(Ke^{at} - b)}$



Geometric Techniques

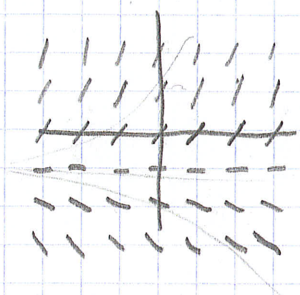
$y' = f(t,y) \quad y(t_0) = y_0$

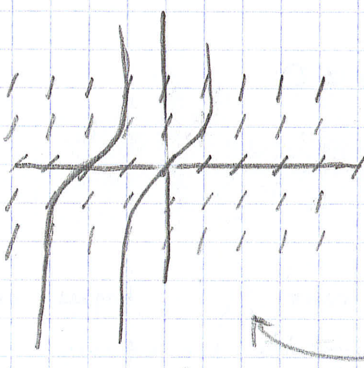


Solution Curve unknown, but slope at (t_0, y_0) is known

Direction Field

Ex. $y' = 1+y$ $\qquad$ $y' = 1+y^2$





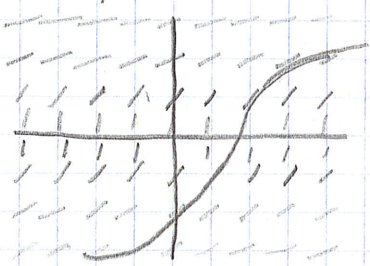
$\frac{dy}{dt} = \frac{1}{1+y^2}$

$\int \frac{1}{1+y^2}\, dy = t + c$

$\tan^{-1}(y) = t + c$

$\boxed{y = \tan(t+c)}$

$y' = \frac{1}{y^2} \quad (y \neq 0)$

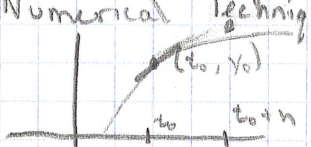


$\int \frac{dy}{dt} y^2\, dt = \int 1\, dt$

$\frac{1}{3} y^3 = t + c$

$y = (3t + 3c)^{1/3}$

Numerical Techniques:



$y' = f(t,y)$

$t_1 = t_0 + h$ (where h is small)

$y_0 + f(t_0, y_0) h = y_1$

$y_n = y_{n-1} + h f(t_{n-1}, y_{n-1})$

Euler Method

Qualatative Technique:

$y' = 1 + y^2 \quad y(0) = 1$

$1 \leq 1 + y^2 \leq 2y^2$

$y' = 1$ $\qquad$ $y' = 2y^2$

$y(0) = 1$ $\qquad$ $y(0) = 1$

$y = t + 1$ $\qquad$ $\tan^{-1} y = t + c$

$t + 1 \leq y = g(t) \leq \frac{1}{1-2t}$