

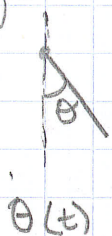
1.  $y' = ry$      $y = y(t)$

Applications: Rate of change of bacteria in medium is directly proportional to the amount present  
 Radioactive decay  
 Continuous compounding

2.  $m \frac{dy}{dt} = ma = F = -mg$  (Falling body without air resistance)

With resistance:  $m \frac{dy}{dt} = -mg + \gamma v$  ,  $\gamma > 0$

3. Rigid Pendulum



$$\frac{d^2\theta}{dt^2} + \frac{g}{L} \sin\theta = 0$$

Note: The "order" of a diff. equation refers to the highest order of the derivatives that appear.  
 • Diff. equations of 1 unknown variable are 'ordinary'  
 • Above are explicit diff. equations -  $\frac{dy}{dt}$  appears by itself

First Order Ordinary Differential Equations (explicit)

$$y' = f(t, y)$$

1.  $y = Ke^{rt} \Rightarrow y' = re^{rt} = ry$   
 K any constant

Linear Equations with constant coefficients:

$y = y(t)$   
 $y' = ay + b$      $a, b$  constants

$$\frac{dy}{dt} = ay + b$$

$a = 0; \frac{dy}{dt} = b \rightarrow y = bt + c$

$a \neq 0; \frac{dy}{dt} = ay + b$

$$\int \frac{dy}{dt} \cdot \frac{dt}{ay+b} = \int 1 dt = t + c$$

$$\frac{1}{a} \ln|ay+b| = t + c \rightarrow \ln|ay+b| = a(t+c)$$

$$ay+b = \pm e^{\frac{ac}{a}} e^{at}$$

$$|ay+b| = e^{\frac{ac}{a}} e^{at}$$

$$ay = Ke^{at} - b \rightarrow y = \frac{1}{a} (Ke^{at} - b)$$

