Dio Margetis
dio@math
Math 4416

* www.math.umd.edu/~dio/courses/MAT
teaching → Math 241

Calculus Aug 30

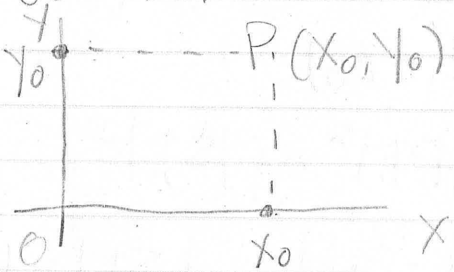
Rm 3304 Math
T.A. office

Reading → 11.1, 11.2

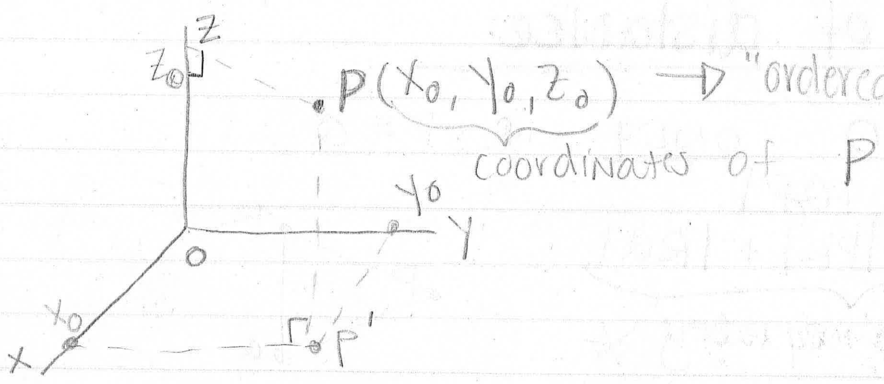
- 4 exams, lowest dropped
- 8 quizzes, lowest dropped

Vectors

11.1 COORDINATE SYSTEMS



3-dimensional space



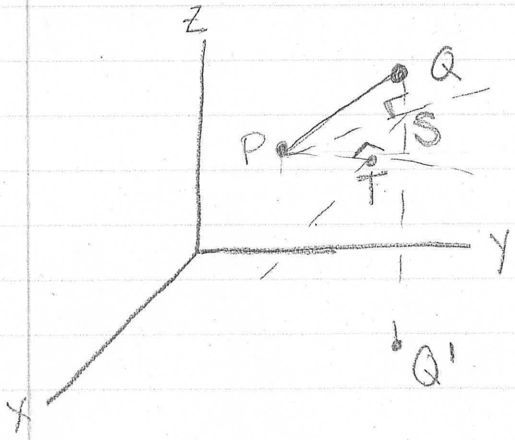
The axis x, y, z separate space into 8 parts: octants

(1st octant all coordinates are positive)

Consider $P = (x_0, y_0, z_0)$ & $Q(x_1, y_1, z_1)$

Distance between P & Q:

$$\underbrace{|PQ|}_{\text{means "distance b/w P \& Q"}} = \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2}$$



$(|x_1 - x_0|^2 + |y_1 - y_0|^2)(|z_1 - z_0|^2)$

$$|PQ|^2 = |PS|^2 + |SQ|^2$$

$$|SQ| = |z_1 - z_0|$$

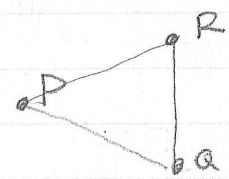
$$|PS|^2 = |PT|^2 + |ST|^2$$

$$|PT| = |y_1 - y_0| \qquad |ST| = |x_1 - x_0|$$

3 Laws of distance:

- $|PQ| = 0$ ONLY if $P = Q$
- $|PQ| = |QP|$
- *KNOW* $|PQ| \leq |PR| + |RQ|$

triangle inequality

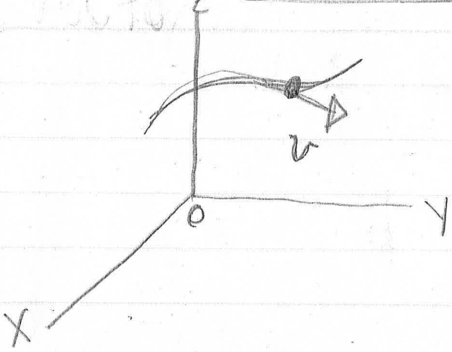


DEFINITION 1: Sphere with center $P_0 = (x_0, y_0, z_0)$ and Radius a = Set of all points (x, y, z) that satisfy

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = a^2$$

Saying all points on a sphere are same distance from center (a)

11.2 Vectors in Space

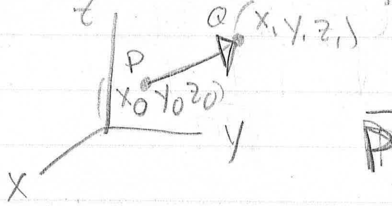


Velocity: magnitude (length of arrow) → speed
direction

→ vector

Initial PT = (x_0, y_0, z_0)
Terminal PT = (x_1, y_1, z_1)

Change in coordinates:
$(x_1 - x_0)$
$(y_1 - y_0)$
$(z_1 - z_0)$



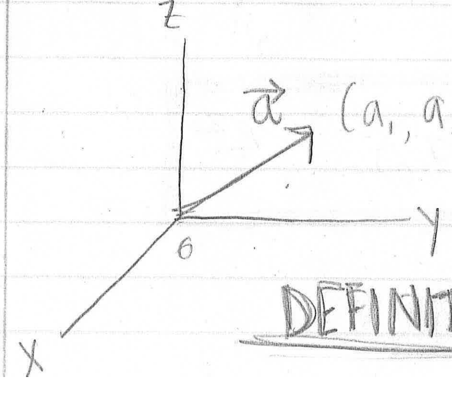
$\overrightarrow{PQ} = (x_1 - x_0,\ y_1 - y_0,\ z_1 - z_0)$

Vector is defined by $(x_1 - x_0,\ y_1 - y_0,\ z_1 - z_0)$

Definition 2: A vector is the ordered triplet (a_1, a_2, a_3) of real numbers. (a_1, a_2, a_3 = components)

* 2 vectors are equal if & ONLY if their components are equal!

vectors (a_1, a_2, a_3) & (b_1, b_2, b_3) are equal if & only if $a_1 = b_1$, $a_2 = b_2$, $a_3 = b_3$!!



$\vec{a}$ (a_1, a_2, a_3) usually take initial pt @ O

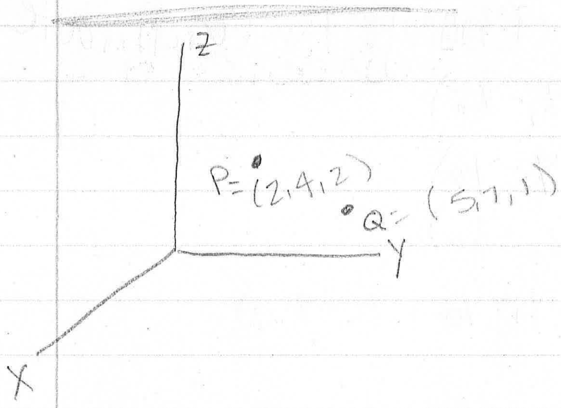
Definition 3: Length (or Norm) of $\vec{a} = (a_1, a_2, a_3)$ is $\|\vec{a}\| = \sqrt{a_1^2 + a_2^2 + a_3^2}$

DEFINITION 4: $\vec{a}=(a_1, a_2, a_3)$, $\vec{b}=(b_1, b_2, b_3)$

THEN: $\vec{a} \pm \vec{b} = (a_1 \pm b_1, a_2 \pm b_2, a_3 \pm b_3)$

c=real # $\quad c\vec{a} = (ca_1, ca_2, ca_3)$

EXAMPLES:



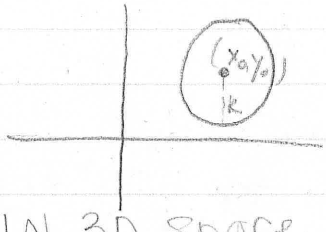
What is the distance between P & Q

$$|\vec{PQ}| = \sqrt{(5-2)^2+(7-4)^2+(1-2)^2}$$
$$= \sqrt{9+9+1}$$
$$\sqrt{19}$$

14. Show that
$$x^2+y^2+z^2+6x-8y-4z+4=0$$
is an equation of a sphere

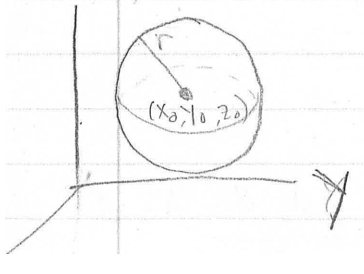
o o o o o

IN the plane, a circle with center (x_0, y_0) & radius R has equation $(x-x_0)^2+(y-y_0)^2=R^2$



IN 3D space, the equation of the sphere w/ radius R & center (x_0, y_0, z_0) is

$$\boxed{(x-x_0)^2+(y-y_0)^2+(z-z_0)^2=R^2}$$



* UNIT VECTORS *

$\hat{i} = (1,0,0)$
$\hat{j} = (0,1,0)$ } IN a space / plane
$\hat{k} = (0,0,1)$

14. Show that

$$x^2+y^2+z^2+6x+8y-4z+4=0$$

is an equation of a sphere

$$x^2+6x+\underline{9}+y^2+8y+\underline{16}+z^2-4z+4=\underline{9}+\underline{16}$$

complete the square

$$(x+3)^2+(y+4)^2+(z-2)^2=25$$

center: $(-3,-4,2)$
radius: 5

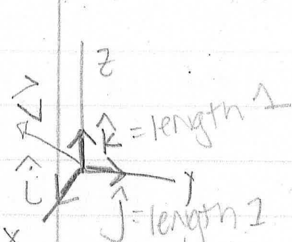
EX: find the equation of the open ball w/ radius 7 & center $(-1,2,0)$

$$(x+1)^2+(y-2)^2+z^2<7^2$$
$$(x+1)^2+(y-2)^2+z^2<49$$

11.2 Vectors practice

* length of vector is just the magnitude

ex: find the length of $\vec{v}=\hat{i}-\hat{j}+\hat{k}$ $\quad |\vec{v}|=\sqrt{\hat{i}^2-\hat{j}^2+\hat{k}^2}$



$$|\vec{v}|=\sqrt{1^2+(-1)^2+1^2}$$
$$=\sqrt{3}$$

UNIT VECTORS: EX
find a unit vector parallel to $\vec{v}$

$$\frac{\vec{v}}{|\vec{v}|}=\frac{1}{\sqrt{3}}\hat{i}-\frac{1}{\sqrt{3}}\hat{j}+\frac{1}{\sqrt{3}}k$$

a unit vector is a vector, $\vec{u}$ where the length of $\vec{u}=1$. NOT necessarily $\hat{i},\hat{j},\&\,\hat{k}=1$