

MWF 11-11:50am - AM 0131
Tu/Th 12-12:50pm - MH 0306

sect 0251

MATH 161

Dr. R. Johnson
2107

web Assign

<http://www.math.umd.edu/wers/rj/m161/08.htm>

on web Assign:

instead of

write

e^{ax}

$\exp(ax)$

\arctan

atan

∞

Infinity

Integrals:

$$\int_a^b f'(t) dt = f(b) - f(a)$$

$$\int_a^b f' = f + \underline{C} \quad (\text{web Assign wants the constant})$$

Riemann Sum:

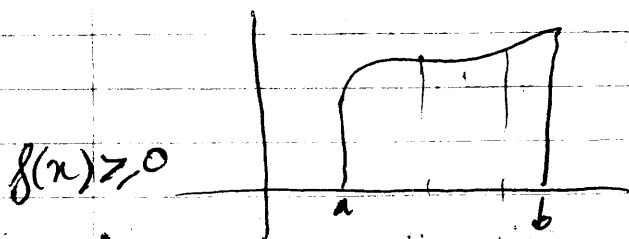
f on $[a, b]$

$$P = \{a = x_0 < x_1 < \dots < x_n\}$$

$$t_i \in [x_{i-1}, x_i]$$

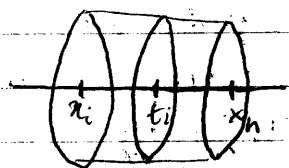
$$\int_a^b f(x) dx = \frac{b-a}{n} \sum_{i=1}^n f(x_i^*)$$

Regular partition $[a, b]$ is



$$\int_a^b f = \text{area under graph}$$

$$D = \{a \leq x \leq b, A(x) \text{ cross-section}\}$$



$$x_{i-1} \leq t_i \leq x_i, A(t_i)$$

$$A(x) \approx A(t_i)$$

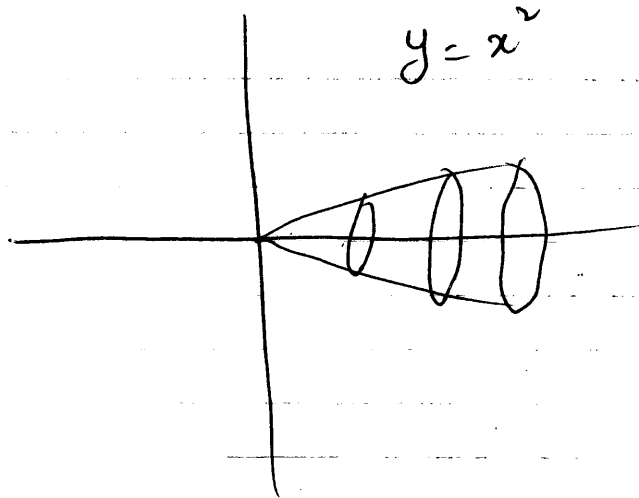
$$\Delta V_i = A(x) (x_i - x_{i-1}) = A(t_i) (x_i - x_{i-1})$$

Definit: If a volume lies on $a \leq x \leq b$ & at each x in cross-sectional area $A(x)$, then $V = \int_a^b A(x) dx$.

Cylindrical height h ,

$$A(x) = \pi r^2, a \leq x \leq b$$

$$V = \int_a^b \pi r^2 dx = \pi r^2 h$$



Cross sectional x
 $0 \leq x \leq 1$ is a circle
 radius x^2
 $A(x) = \pi (x^2)^2$
 $= \pi x^4$

$$V = \int_0^1 \pi x^4 dx = \pi \frac{x^5}{5} \Big|_0^1$$

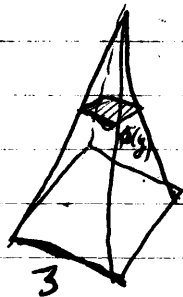
$$= \frac{\pi}{5}$$

If cross-sectional area are known at each y.

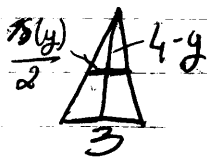
$$V = \int_c^d A(y) dy$$

Find the volume of a pyramid with a square base of length 3 and height 4.

$s(y) = \text{side length}$



$$0 \leq y \leq 4$$



$$\frac{s(y)}{4-y} = \frac{3}{4} \Rightarrow s(y) = \frac{3(4-y)}{4} = 3 \left(\frac{4-y}{4} \right)$$

$$A(y) = (r(y))^2$$

$$= \frac{3}{4} (4-y)^2$$

$$V = \frac{9}{16} \int_0^4 (4-y)^2 dy$$

$$\frac{r(y)}{h-y} = \frac{b/2}{h} \rightarrow \frac{r(y)}{b} = \frac{h-y}{h}$$

$$r(y) = b \left(\frac{h-y}{h} \right)$$

$$V = \int_0^h \frac{b^2 (h-y)^2}{h^2} dy$$

$$= \frac{1}{3} b^2 h$$

$$= \frac{b^2}{h^2} \int_0^h (h^2 - 2hy + y^2) dy$$

Circle radius $f(x)$

Even fct $f(-x) = f(x)$

$$A(x) = \pi (f(x))^2$$

then

$$\int_{-n}^n f = \int_0^n + \int_{-n}^0 = \int_0^n + \int_0^n$$

$$V = \int_a^b \pi f(x)^2 dx$$

$$= 2 \int_0^n f$$

$$x^2 + y^2 = r^2$$

$$y = \sqrt{r^2 - x^2} \quad -r \leq x \leq r$$

uneven

$$f(-x) = -f(x)$$

then

$$\int_{-n}^n f(x) dx = 0$$

$$V = \int_{-n}^n A(x) dx$$

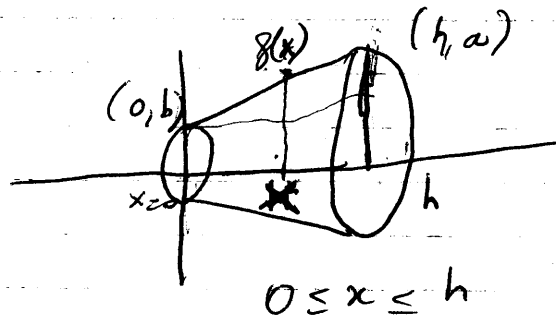
$$= 2\pi \left(n^2 x - x^3/3 \right) \Big|_0^n$$

$$= \int_{-n}^n \pi (\sqrt{r^2 - x^2})^2 dx$$

$$= 2\pi \cdot \frac{2}{3} r^3 = \frac{4\pi r^3}{3}$$

$$= \pi \int_{-n}^n (r^2 - x^2) dx = \pi r^2 (r^2 - x^2) \Big|_{-n}^n$$

Frustrum of a cone:



b & a are radii of b

$$\frac{b - f(x)}{0 - x} = \frac{b - a}{0 - h}$$

$$\frac{f(x) - b}{x} = \frac{a - b}{h}$$

$$f(x) = b + \left(\frac{a-b}{h}\right)x$$

$$V = \pi \int_0^h \left(b + \frac{a-b}{h}x\right)^2 dx$$

$$u = b + \frac{a-b}{h}x$$

$$du = \frac{a-b}{h} dx$$

$$V = \pi \int_b^a u^2 \frac{h}{a-b} du$$

$$= \frac{\pi h}{a-b} \left(\frac{a^3 - b^3}{+3}\right) = \frac{\pi h}{3} \left[a^3 \left(\frac{a^3 - b^3}{a-b}\right) \right]$$

$$V = \pi \int_0^3 9 - x^2$$

$$V = \pi \left[9x - \frac{1}{3}x^3 \right] \Big|_0^3$$

$$V = \pi \left[27 - \frac{1}{3}(27) - 0 \right]$$

$$V = \pi [27 - 9]$$

$$V = \pi [18]$$

$$V = \pi \int x^3 \sqrt{x^3 + 1}$$

=

$$\frac{x^3 (x^3 + 1)^{-3}}{3}$$

$$x (x^3 + 1)^{-3}$$

$$(g(f(x)))' = g'(f(x)) \cdot f'(x)$$

$$u (u^3 + 1)^{-3}$$

$$3x^2 \cdot \frac{1}{4} x^4$$

$$\int x^3 = \frac{1}{4} x^4$$

$$\int x = \frac{1}{2} x^2$$

$$f'(g(x)) \cdot g'(x)$$

$$f'(g(x)) = (x^3 + 1)^{-3}$$

$$g'(x) = x$$

$$g(x) = \frac{1}{2} x^2$$

$$f(x) =$$

$$\frac{-1}{2} (x^3 + 1)^{-2}$$

$$\int x (x^3 + 1)^{-3}$$

$$f'(g(x)) \cdot g'(x)$$

$$\frac{-1}{2} x^{+2} = g(x)$$

$$f'(g(x)) = (x^3 + 1)^{-3}$$

<http://www.weassign.net/und/login.html>

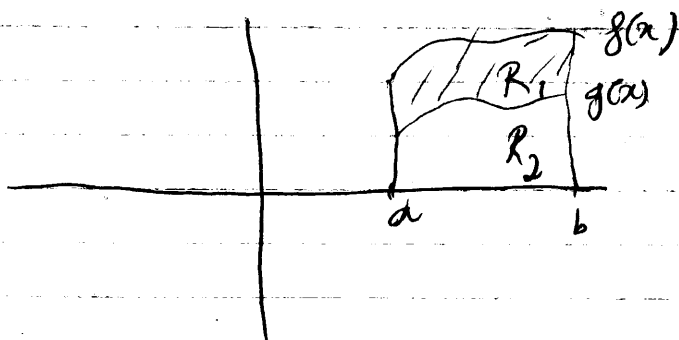
$$2) V = \int A(x) dx$$

Rotate region below graph of $f(x) \geq 0$ about x -axis

$$A(x) = \pi f(x)^2$$

$$V = \pi \int_a^b f(x)^2 dx$$

$$R = \{(x, y) \mid a \leq x \leq b, 0 \leq g(x) \leq y \leq f(x)\}$$



$$R_1 = \{(x, y) \mid a \leq x \leq b, 0 \leq y \leq f(x)\} \quad V_{R_1} = \pi \int_a^b f(x)^2 dx$$

$$R_2 = \{(x, y) \mid a \leq x \leq b, 0 \leq y \leq g(x)\}$$

$$V_{R_2} = \pi \int_a^b g(x)^2 dx$$

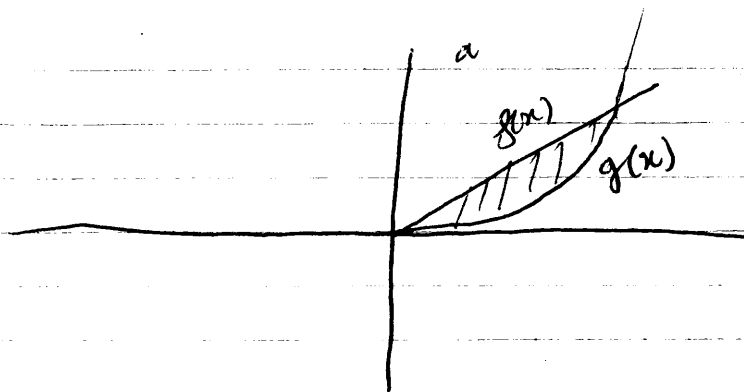
$$V_R = V_{R_1} - V_{R_2}$$

$$\text{So } V = \pi \int_a^b [f(x)^2 - g(x)^2] dx$$

19/05/2008

math 141

until Monday
on the 1st
homework



$$f(x) = 5x$$

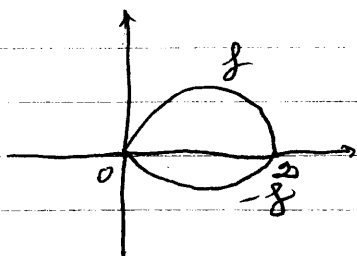
$$g(x) = x^2 \quad 0 \leq x \leq 5 \quad \quad \quad 0 \leq x \leq 5$$

$$V = \pi \int_0^5 (5x)^2 - (x^2)^2 dx = \pi \int_0^5 (25x^2 - x^4) dx$$

$$R = \{ (x, y) \mid 0 \leq x \leq 10, \text{ |of graphs of } f \text{ \& } g \}$$

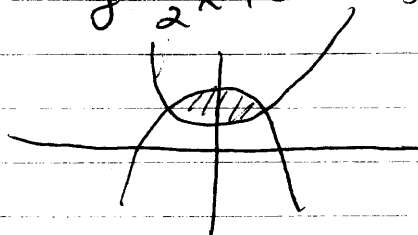
$$V_1 = \pi \int_0^5 (25x^2 - x^4) dx + \pi \int_5^{10} [(x^2)^2 - (25x)^2]$$

Ex: Find volume of region |of graphs of $f(x) = 2x - x^2$ & $g(x) =$
 $g(x)$



$$V = \pi \int_0^2 [2x - x^2]^2 dx$$

H 15) $y = \frac{1}{2}x^2 + 3$ & $y = 12 - \frac{1}{2}x^2$



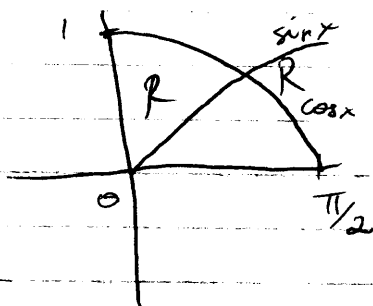
$$\frac{1}{2}x^2 + 3 = 12 - \frac{1}{2}x^2$$

$$x^2 = 9 \quad \Rightarrow \quad x = \pm 3$$



$$\begin{aligned}
 V &= \pi \int \left[\left(12 - \frac{1}{2}x^2\right)^2 - \left(\frac{1}{2}x^2 + 3\right)^2 \right] dx \\
 &= \pi \int_{-3}^3 15(9 - x^2) dx
 \end{aligned}$$

eg: Rotate neg. | graphs of $\cos x$ & $\sin x$ on $[0, \frac{\pi}{2}]$



$$\begin{aligned}
 \cos x &= \sin x \\
 x &= \frac{\pi}{4}
 \end{aligned}$$

$$\begin{aligned}
 V &= \pi \int_0^{\pi/4} [\cos^2 x - \sin^2 x] dx + \int_{\pi/4}^{\pi/2} [\sin^2 x - \cos^2 x] dx \\
 &= \pi \int_0^{\pi/4} \cos 2x dx + \int_{\pi/4}^{\pi/2} -\cos 2x dx = \pi \left[\frac{1}{2} \sin 2x \right]_0^{\pi/4} - \left[\frac{1}{2} \sin 2x \right]_{\pi/4}^{\pi/2}
 \end{aligned}$$

→ Graph $y = f(x)$

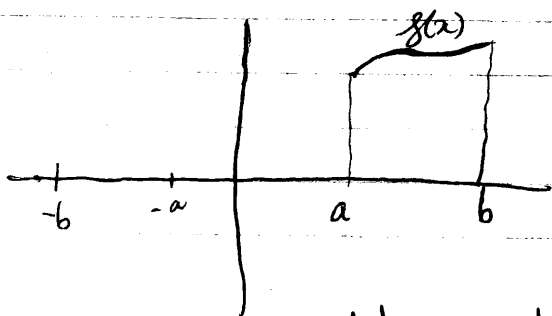
$$R = \{(x, y) \mid a \leq x \leq b, 0 \leq y \leq f(x)\}$$

$$V = \pi \int_a^b (f(x)^2 - g(x)^2) dx \quad (\text{rotate about } x\text{-axis})$$

$$V = \pi \int_c^d (f(y)^2 - g(y)^2) dy$$

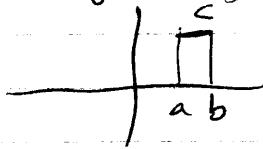
(rotate about y-axis)

$$R = \{(x, y) \mid c \leq y \leq d, g(x) \leq x \leq f(y)\}$$



rotate around the x-axis

In case of a straight line



when rotating around y-axis, we get a cylinder

$$V = \pi b^2 c - \pi a^2 c$$

$$V = \pi (b^2 - a^2) c$$

In general,

$$\text{let } P = \{a \leq x_0 \leq x_1 \leq \dots \leq x_n = b\}$$

$$V = \sum \Delta V_k \quad \Delta V_k = \pi f(t_k) (x_k^2 - x_{k-1}^2)$$

$$= \pi \sum_{k=1}^n f(t_k) (x_k^2 - x_{k-1}^2)$$

$$= 2\pi \sum_{k=1}^n f(t_k) \frac{(x_k^2 + x_{k-1}^2)}{2} (x_k - x_{k-1})$$

$$= 2\pi \sum_{k=1}^n (t_k f(t_k)) \frac{x_k + x_{k-1}}{2} (x_k - x_{k-1})$$

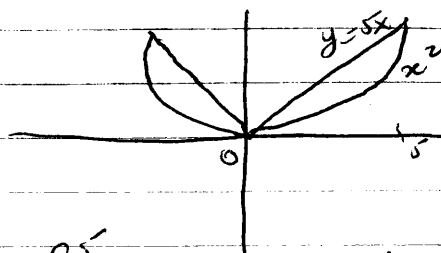
$$V = 2\pi \int_a^b x f(x) dx$$

$$R = \{(x, y) \mid a \leq x \leq b \quad g(x) \leq y \leq f(x)\}$$

$$V = 2\pi \int_a^b x [f(x) - g(x)] dx$$

(rotating around the y-axis)

eg.:



$$0 \leq x \leq 5$$

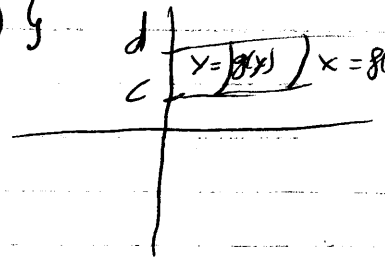
$$V = 2\pi \int_0^5 x [5x - x^2] dx$$

$$= 2\pi \int_0^5 (5x^2 - x^3) dx$$

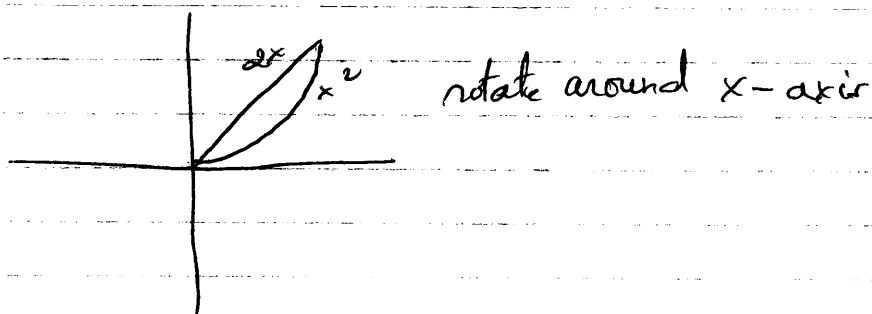
$$R = \{(x,y) \mid c \leq y \leq d, g(y) \leq x \leq f(y)\}$$

$$V = 2\pi \int_a^b x(f(y) - g(y)) dx$$

$$V = 2\pi \int_c^d y[f(y) - g(y)] dy$$



e.g.: Region below graph of $y=2x$ & $y=x^2$, $0 \leq x \leq 2$



$$\textcircled{1} \quad V = \pi \int_0^2 [(2x)^2 - (x^2)^2] dx$$

$$= \pi \int_0^2 (4x^2 - x^4) dx$$

$$= \pi \left[\frac{4}{3}x^3 - \frac{x^5}{5} \right]_0^2$$

$$= \pi \left(\frac{32}{3} - \frac{32}{5} \right)$$

$$V = \frac{64}{15} \pi$$

it doesn't matter what method u use (rotation from x or y-axis) u find the same answer

we want to rotate around the y-axis we find x

$$\textcircled{2} \text{ Another way: } x = \frac{1}{2}y \quad x = \sqrt{y} \quad 0 \leq y \leq 4$$

$$V = 2\pi \int_0^4 y \left[\sqrt{y} - \frac{1}{2}y \right] dy$$

$$= 2\pi \left[\frac{2}{5}y^{5/2} - \frac{y^3}{6} \right]_0^4$$

$$= 2\pi \left[\frac{64}{5} - \frac{64}{6} \right] = 2\pi \left(\frac{64}{15} \right) = \frac{64\pi}{15}$$

09/08/2008

4 methods:

→ Disc method

• rotate about the x-axis $\Rightarrow \pi \int_a^b f(x)^2 dx$
 $y = f(x)$

• rotate about y-axis $\Rightarrow \pi \int_c^d g(y)^2 dy$
 $x = g(y)$

→ Washer method

• rotate about x-axis $\Rightarrow \pi \int_a^b [f(x)^2 - g(x)^2] dx$

$R = \{(x, y) \mid a \leq x \leq b, g(x) \leq y \leq f(x)\}$

• rotate about y-axis

$\Rightarrow \pi \int_c^d [g(y)^2 - h^2] dy$

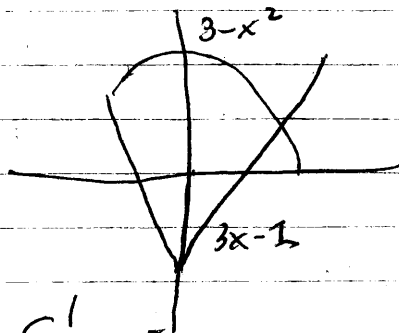
→ Shell method

rotate region R about y-axis

about y-axis

$$V = 2\pi \int_a^b x [f(x) - g(x)] dx$$

e.g. Find V region R bounded by graphs of $y = 3 - x^2$ & $y = 3x - 1$ about y-axis

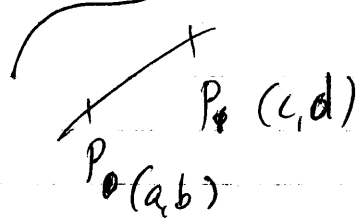


$$\begin{aligned} 3 - x^2 &= 3x - 1 \\ 0 &= x^2 + 3x - 4 \\ (x - 1)(x + 4) \end{aligned}$$

$$V = 2\pi \int_0^1 x [(3 - x^2) - (3x - 1)] dx = 2\pi \int_0^1 [x(4 - 3x - x^2)] dx$$
$$= 2\pi \int_0^1 (4x - 3x^2 - x^3) dx$$

Length of a curve:

$$y = f(x)$$



$$L = \sqrt{(c-a)^2 + (d-b)^2}$$

$$P = \{x_0 = a, x_1, \dots, x_n = b\}$$

$$\Delta x_k = \sqrt{(x_k - x_{k-1})^2 + (f(x_k) - f(x_{k-1}))^2}$$

$$L = \sum_{k=1}^n \Delta x_k = \sum_{k=1}^n \sqrt{(x_k - x_{k-1})^2 + (f(x_k) - f(x_{k-1}))^2}$$

f differentiable

$$f(x_k) - f(x_{k-1}) = f'(m_k)(x_k - x_{k-1})$$

$$L = \sum_{k=1}^n \sqrt{(x_k - x_{k-1})^2 + f'(m_k)^2 (x_k - x_{k-1})^2}$$

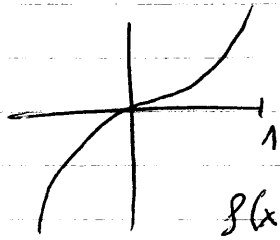
$$= \sum_{k=1}^n \sqrt{1 + f'(m_k)^2} (x_k - x_{k-1}) = R(\sqrt{1 + f'^2}, \{m_k\}, P)$$

Δx_k

$\overset{!}{=} \text{Riemann sum}$

$$\text{Def } L = \int_a^b \sqrt{1 + f'(x)^2} dx$$

e.g: $y = \frac{1}{3}x^3, -1 \leq x \leq 1$



$$f(x) = \frac{1}{3}x^3$$

$$f'(x) = x^2$$

$$L = \int_{-1}^1 \sqrt{1+x^4} dx = 2 \int_0^1 \sqrt{1+x^4} dx \quad \text{Cannot be eval}$$

e.g: $f(x) = 2x+3 \quad 0 \leq x \leq 1$

$(0,3)$ to $(1,5)$ [straight line]

$$\sqrt{(5-3)^2 + (1-0)^2} = \sqrt{5}$$

$$f'(x) = 2$$

$$L = \int_0^1 \sqrt{1+4} = \sqrt{5}$$

e.g: $y = x^{3/2} + 1, 0 \leq x \leq 4$

$$f(x) = x^{3/2} + 1$$

$$f'(x) = \frac{3}{2}x^{1/2}$$

$$L = \int_0^4 \sqrt{1 + \frac{9}{4}x} dx$$

$$u = 1 + \frac{9}{4}x$$

$$du = \frac{9}{4} dx$$

$$L = \int_0^4 u^{1/2} du \times \frac{4}{9}$$

$$L = \frac{4}{9} \int_0^4 u^{1/2} du$$

$$L = \frac{4}{9} \left[\frac{2}{3} u^{3/2} \right]_0^4$$

$$L = \frac{4}{9} \left[\frac{2}{3} (\sqrt{10})^{3/2} - \frac{2}{3} (\sqrt{1})^{3/2} \right]$$

$$L = \frac{4}{9} \left(\frac{2}{3} (\sqrt{10})^{3/2} - \frac{2}{3} \right)$$

$$L = \frac{8}{27} (\sqrt{10})^{3/2} - \frac{8}{27}$$

Sometimes it's impossible even for simple functions there is no integral

ms. 1+

$$y = \ln x - \frac{x^2}{8}, \quad 1 \leq x \leq 2$$

$$f(x) = \ln x - \frac{x^2}{8}$$

$$f'(x) = \frac{1}{x} - \frac{1}{4}x$$

$$L = \int_1^2 \sqrt{1 + \left(\frac{1}{x} - \frac{x}{4}\right)^2} dx$$

$$= \int_1^2 \sqrt{1 + \left(\frac{1}{x^2} - \frac{1}{2} + \frac{x^2}{16}\right)} dx$$

$$= \int_1^2 \sqrt{\frac{1}{x^2} + 1 - \frac{1}{2} + \frac{x^2}{16}} dx$$

$$= \int_1^2 \sqrt{\frac{1}{2} + \frac{1}{x^2} + \frac{x^2}{16}} dx$$

$$= \int_1^2 \sqrt{\left(\frac{1}{x} + \frac{x}{4}\right)^2} dx$$

$$= \int_1^2 \left(\frac{1}{x} + \frac{x}{4}\right) dx$$

$$= \int_1^2 \frac{1}{x} dx + \frac{x}{4} dx$$

$$= \ln x + \frac{x^2}{8} \Big|_1^2$$

$$L = \ln 2 + \frac{3}{8}$$

$$\frac{e^{2x} - 2e^x e^{-x} + e^{-2x}}{4}$$

eg. #10

$$f(x) = \frac{e^x + e^{-x}}{2}, \quad 0 \leq x \leq \ln 2$$

$$f'(x) = \frac{e^x - e^{-x}}{2}$$

$$L = \int_0^{\ln 2} \sqrt{1 + \left(\frac{e^x - e^{-x}}{2}\right)^2} dx$$

$$L = \int_0^{\ln 2} \sqrt{1 + \frac{e^{2x}}{4} + \frac{e^{-2x}}{4} - e^{2x}} dx$$

$$L = \int_0^{\ln 2} \sqrt{1 + \frac{1}{4}e^{2x} - \frac{1}{4} + \frac{e^{-2x}}{4}} dx$$

$$= \int_0^{\ln 2} \sqrt{\frac{1}{4}e^{2x} + \frac{1}{2} + \frac{1}{4}e^{-2x}} dx$$

$$= \int_0^{\ln 2} \sqrt{\left(\frac{e^x + e^{-x}}{2}\right)^2} dx$$

$$= \int_0^{\ln 2} (e^x + e^{-x}) dx$$

$$= \frac{1}{2} [e^x - e^{-x}] \Big|_0^{\ln 2}$$

$$= \frac{1}{2} \left[2 - \frac{1}{2} - (1 - 1) \right] =$$

$$L = \frac{3}{4}$$

52006

sect. 6.4 work

$$W = F \cdot d \quad [\text{Force} \times \text{distance} = \text{Work}]$$

Variable force $F(x)$ $a \leq x \leq b$

$$P = \{ a = x_0 \leq x_1 < \dots < x_n = b \}$$

$$\text{on } [x_{k-1}, x_k], F(x) \approx F(t_k) \quad x_{k-1} \leq t_k \leq x_k$$

$$\Delta W_k \approx F(t_k)(x_k - x_{k-1})$$

$$W = \sum \Delta W_k = \sum$$

read the sectⁿ
on units.

Definitⁿ The work done by a variable force $F(x)$, a is $W = \int_a^b F(x) dx$

e.g. Suppose a leaking wheelbarrow push 100 meters
 $F(x) = 60 \left(1 - \frac{x^2}{20,000} \right)$ $0 \leq x \leq 100$ / [the force we

$$F(0) = 60$$

$$W = \int_0^{100} 60 \left(1 - \frac{x^2}{20,000} \right) dx$$
$$= \int_0^{100} \left(60 - \frac{60x^2}{20,000} \right) dx$$

$$= 60 \left(x - \frac{x^3}{60,000} \right) \Big|_0^{100}$$

=

$$F = m a(x) = m \frac{dv}{dt} = m \frac{dv}{dx} \cdot \frac{dx}{dt} = m \frac{dv}{dx} v$$

$$\int F = \int (m v \frac{dv}{dx}) dx = \int m v dv = \frac{1}{2} m v_1^2 - \frac{1}{2} m v_0^2$$

The weight of the water is 62.5 pounds per cubic foot
 \Rightarrow the weight of the water is $62.5V$

So $W = (\text{weight}) \times (\text{distance}) = 62.5Vd$
 for the work done by the lifting force when pumping water out of a tank & up to a certain level.

If the liquid is another liquid \neq water, 62.5 will be replaced by the weight of 1 cubic foot of the other liquid.

$$W = \int_a^b 62.5 (l-x) A(x) dx$$

Gravitational force $F = -\frac{GMm}{r^2}$

$$W(b) = \int_R^b -\frac{GMm}{r^2} dr = \frac{GMm}{r} \Big|_R^b$$

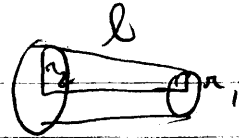
$$W(b) = GMm \left(\frac{1}{b} - \frac{1}{R} \right) = \left(\frac{1}{2} m (v(b))^2 - \frac{1}{2} m (v(R))^2 \right)$$

(work done when spaceship travels a distance b from the ^{center of} earth _{surface})

= radius of the earth.

09/10/2008

A cylinder = $2\pi rh$

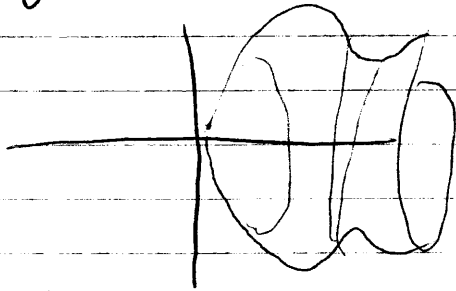


$$L = \int_a^b \sqrt{1 + f'(x)^2} dx$$

$$A = 2\pi \left(\frac{r_1 + r_2}{2}\right) l$$

$$A = \pi(r_1 + r_2) l$$

$y = f(x)$ rotate around x-axis



$$P = \{a = x_0 < x_1 < \dots < x_n = b\}$$

on $[x_{k-1}, x_k]$

$$\Delta S_k = \pi [f(x_{k-1}) + f(x_k)] \Delta x_k$$

$$\Delta S_k = \pi [f(x_{k-1}) + f(x_k)] \sqrt{(x_k - x_{k-1})^2 + (f(x_k) - f(x_{k-1}))^2}$$

$$= \pi [f(x_{k-1}) + f(x_k)] \sqrt{1 + f'(t_k)^2} (x_k - x_{k-1})$$

$$= \pi 2 f(t_k) \sqrt{1 + f'(t_k)^2} \Delta x_k$$

S = Surface Area.

$$\text{Def. } S = 2\pi \int_a^b f(x) \sqrt{1 + f'(x)^2} dx$$

eg: $y = x^3$ on $[0, 1]$

$$S = 2\pi \int_0^1 x^3 \sqrt{1 + (3x^2)^2} dx$$

$$= 2\pi \int_0^1 x^3 \sqrt{1 + 9x^4} dx$$

$$= 2\pi \int_0^1 u^{\frac{1}{2}} \frac{du}{36}$$

Let $u = 1 + 9x^4$
 $du = 36x^3 dx$

$$= \frac{\pi}{18} \left[\frac{2}{3} u^{\frac{3}{2}} \right]_0^1$$

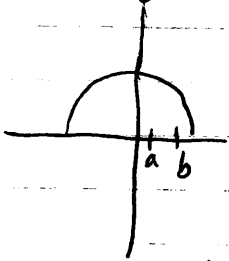
$$= \frac{\pi}{18} \left(\frac{2}{3} \sqrt{1 + 9(1)^4} \right)$$

$$= \frac{\pi}{9} \sqrt{10} \approx 1.1$$

e.g.: $x^2 + y^2 = 1$ (Circle)

$$y^2 = 1 - x^2$$

$$y = \sqrt{1 - x^2}$$



$$-1 \leq a \leq b \leq 1$$

$$f(x) = (1 - x^2)^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2} (1 - x^2)^{-\frac{1}{2}} (-2x)$$

$$= \frac{-x}{\sqrt{1 - x^2}}$$

$$1 + (f'(x))^2 = 1 + \frac{x^2}{1 - x^2} = \frac{1}{1 - x^2}$$

$$S = 2\pi \int_a^b (1 - x^2)^{\frac{1}{2}} \frac{1}{\sqrt{1 - x^2}} dx$$

$$= 2\pi (b - a)$$

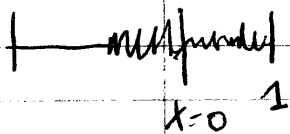
Sphere of radius 1 has surface area 4π
 $= 2\pi [1 - (-1)]$

Hook's law: Restoring force exerted by a spring stretching x units from equilibrium is

$k = \text{spring constant}$

$$F(x) = kx$$

e.g. Work required to stretch a spring 2 unit from equilibrium is 10^6 ergs. Find the amount of work required to stretch to 3 units from equilibrium.



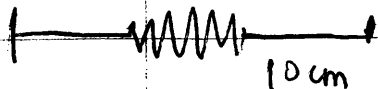
$$W = \int_0^2 kx \, dx = 10^6 \quad 10^6 = \int_0^2 kx \, dx$$

$$W_1 = \int_1^3 kx \, dx \quad k = 2 \times 10^6$$

$$= 2 \times 10^6 \int_1^3 x \, dx$$

$$= 2 \times 10^6 \left(\frac{1}{2} x^2 \right) \Big|_1^3 = 8 \times 10^6$$

e.g. $k = 8 \times 10^5$ dynes/cm, 10 cm from equilibrium
~~10 cm~~ $F = 4 \times 10^6$ dynes
 Additional 10 cm - w?



$$F = kx \Rightarrow k = 4 \times 10^5$$

$$W = \int_{10}^{20} (4 \times 10^5) x \, dx$$

Work done in moving volume of

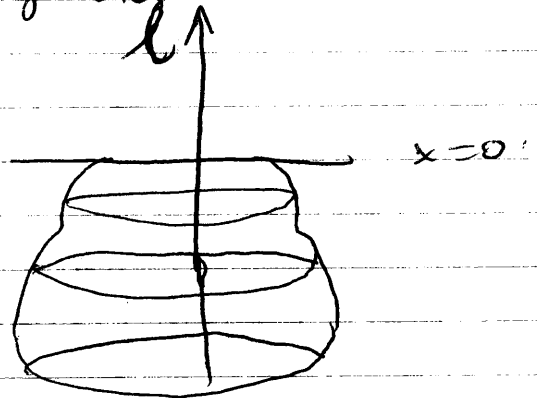
Density of water = 62.5 lb/ft^3



move a volume of water V a distance d .

$$W = Fd = (62.5V) d.$$

pool of water



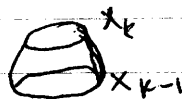
$$a \leq x \leq b \leq 0$$

$$P = \{a = x_0 < x_1 < x_2 < \dots < x_n = b\}$$

$$[x_{k-1}, x_k] \quad x \in [x_{k-1}, x_k]$$

$$d = l - x_k, \quad x_{k-1} \leq t_k \leq x_k$$

$$\Delta V_k = A(t_k) (x_k - x_{k-1})$$



$$\Delta V_k = \int_{x_{k-1}}^{x_k} A(x) \, dx \approx A(t_k) (x_k - x_{k-1})$$

7.5.1

$$\Delta W_k = 62.5 A(t_k) (l - t_k) (x_k - x_{k-1})$$

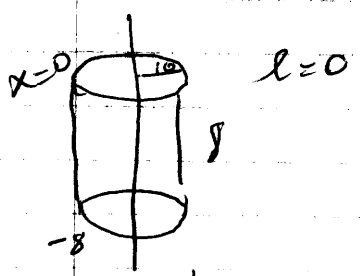
$$W = \sum_{k=1}^n \Delta W_k$$

$$= 62.5 \sum_{k=1}^n P(A(x)(l-x), P, \{t_k\})$$

Def.

$$W = 62.5 \int_a^b A(x)(l-x) dx.$$

e.g.

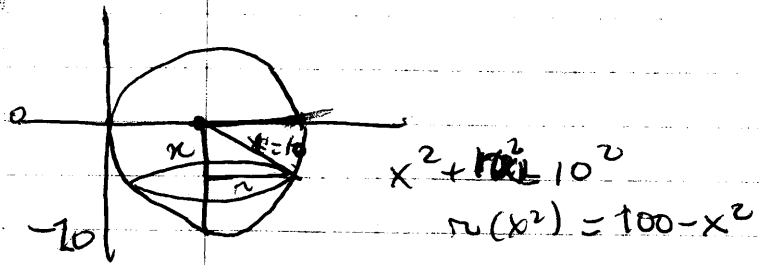


we want to pull all the water out.

$$W = 62.5 \int_{-8}^0 (0-x)(100\pi)$$

$$A(x) = \pi(10)^2 = 100\pi$$

e.g. the pool is hemispherical



$$W = \int_{-10}^0 62.5 (0-x) (\pi(100-x^2)) dx$$

$$= 62.5 \int_{-10}^0$$

(l is defined as the height of the solid water in the solid)

$$W = \int F(x) dx$$

$$F(x) = m a(x) = m \frac{d^2 x}{dt^2} = m \frac{dv}{dt}$$

$$= m \frac{dv}{dx} \times \frac{dx}{dt}$$

$$= m \frac{dv}{dx} \times v$$

$$W = \int_a^b F(x) dx$$

$$= \int_a^b m v \frac{dv}{dx} dx$$

$$= \frac{1}{2} m v^2 \Big|_a^b$$

$$= \frac{1}{2} m v(b)^2 - \frac{1}{2} m v(a)^2$$



$$F(r) = G \frac{Mm}{r^2}$$

$$W = \int_R^b F(r) dr = \int - \frac{GMm}{r^2} dr = + GMm \left. \frac{1}{r} \right|_R^b$$

$$= GMm \left(\frac{1}{b} - \frac{1}{R} \right)$$

$$\frac{1}{2} m v(b)^2 - \frac{1}{2} m v(R)^2 = GMm \left(\frac{1}{b} - \frac{1}{R} \right)$$

$$\frac{1}{2} m v(b)^2 - GMm/b = \frac{1}{2} m v(R)^2 - \frac{GMm}{R}$$

What is the escape velocity?

$$\lim_{b \rightarrow \infty} v(b) = 0$$

$$\frac{1}{b} \rightarrow 0$$

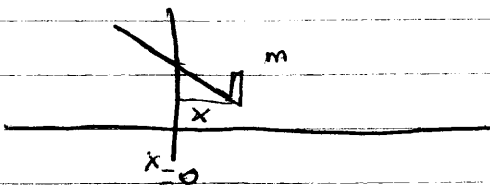
$$\text{LHS} \rightarrow 0$$

$$0 = \frac{1}{2} m v(R)^2 - \frac{GMm}{R}$$

$$v(R)^2 = \frac{2GM}{R}$$

$$v(R) = \sqrt{2GM/R}$$

Moment & center of mass:



~ weight (m, x)

$$M_y = mx$$

$$M_y = m_1 x_1 + m_2 x_2 + \dots + m_n x_n = m \bar{x}$$

Center of mass (\bar{x}, \bar{y})

$$\bar{x} = \frac{m_1 x_1 + m_2 x_2 + \dots + m_n x_n}{m_1 + m_2 + \dots + m_n}$$

$$m = m_1 + m_2 + \dots + m_n$$

$$\bar{y} = \frac{m_1 y_1 + m_2 y_2 + \dots + m_n y_n}{m_1 + m_2 + \dots + m_n}$$

e.g: $m_1 = 2(1, 2)$

$$m_2 = 4(-2, 1)$$

$$m_3 = 5(-1, -2)$$

$$m_4 = 7(0, 3)$$

$$m = 7 + 5 + 4 + 2 = 18$$

$$M_y = 2 \cdot 1 + 4(-2) + 5(-1) + 7(0) = -15$$

$$M_x = 2 \cdot 2 + 4 \cdot 1 + 5(-2) + 7(3) = 12$$

$$\bar{x} = -15/18 = -5/6 = -M_y/m$$

$$\bar{y} = 12/18 = 2/3 = M_x/m$$