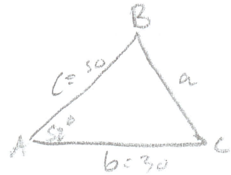
eg. Solve the Oblique triangle w/ $A = 52^\circ$, $b = 30cm$, $c = 50cm$ (SAS)



① use Law of cosines to Find missing side.

$a^2 = b^2 + c^2 - 2bc\cos A$

$= (30)^2 + (50)^2 - 2(30)(50)\cos(52)$

$= 1552 \rightsquigarrow a = 39.4$

② use Law of Sines to Find another angle

$\frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow \sin B \frac{a}{\sin A} = b \rightarrow \sin B = \frac{b\sin A}{a} = \frac{30\sin 52^\circ}{39.4} = .6$

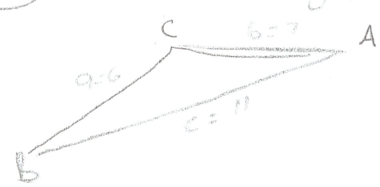
$\sin B = .6 \longrightarrow B = \sin^{-1}(.6) = \boxed{36.9^\circ}$ or $\cancel{B = 180 - 36.9^\circ = 143.1^\circ}$

③ use the Fact that angle sum to 180°

$A + B + C = 180^\circ$

$C = 180^\circ - (A+B) = 180^\circ - (52^\circ + 36.9^\circ) = 180 - (88.9^\circ) = \boxed{91.1^\circ}$

eg. Solve the triangle w/ $a=6$, $b=7$, $c=11$ (SSS)



① use the Law of cosines to find the biggest angle (opposite to the largest side)

- In this case, C.

$c^2 = a^2 + b^2 - 2ab\cos C$

$2ab\cos C = a^2 + b^2 - c^2$

$\cos C = \frac{a^2+b^2-c^2}{2ab} = \frac{6^2+7^2-11^2}{2\cdot 6\cdot 7}$

$= \frac{36+49-121}{84} = \frac{85-121}{84} = \frac{-36}{84} = -\frac{3}{7}$

$C = \cos^{-1}(-\frac{3}{7}) = 115.4^\circ$

② $\frac{c}{\sin C} = \frac{a}{\sin A} \Rightarrow \sin A = \frac{a\sin C}{c} = \boxed{\frac{6\sin\left(\frac{2\sqrt{10}}{7}\right)}{11}}$

$A = \sin^{-1}(\ ) = 29.8^\circ$

$\sin C = \sqrt{1-\cos^2 C}$

$= \sqrt{1 - \frac{3^2}{7^2}} = \frac{\sqrt{40}}{7} = \frac{2\sqrt{10}}{7}$

$B = 180 - (A+C) = 34.8^\circ$

$\text{Area} = \frac{1}{2}ab\sin C$

$= \frac{1}{2}ac\sin B$

$= \frac{1}{2}bc\sin A$

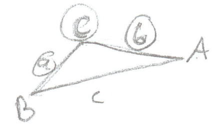
$\boxed{\text{Heron's Formula}}$

$s = \frac{1}{2}(a+b+c)$

$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$

a A
b B
c C

Laws of Sines: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$



Law of cosines:

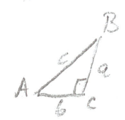
$a^2 = b^2 + c^2 - 2bc\cos A$

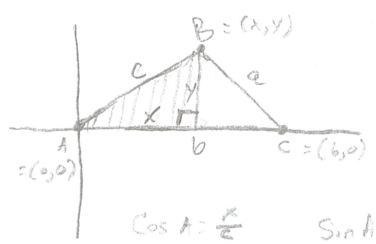
$b^2 = a^2 + c^2 - 2ac\cos B$

(*) $c^2 = a^2 + b^2 - 2ab\cos C$

$(1^{st}\text{ side})^2 = (2^{nd}\text{ side})^2 + (3^{rd}\text{ side})^2 - 2(2^{nd}\text{ side})(3^{rd}\text{ side})\cos(1^{st}\text{ angle})$

If one of the angles is $90°$, say $C = 90°$, then $\cos C = 0$, so (*) becomes
$c^2 = a^2 + b^2$





$\cos A = \frac{x}{c}$ $\sin A = \frac{y}{c}$

$\boxed{x = c\cos A}$ $\boxed{y = c\sin A}$

a = distance from $B = (x, y)$ to $C = (b, 0)$

$= \sqrt{(x-b)^2 + (y-0)^2}$ (Distance Formula)

$a^2 = (x-b)^2 + y^2$ (x- & y-coord in terms of cos & sin)

$= (c\cos A - b)^2 + (c\sin A)^2$

$= (c\cos A)^2 - 2bc\cos A + b^2 + c^2\sin^2 A$

$= c^2\cos^2 A + c^2\sin^2 A + b^2 - 2bc\cos A$

$= c^2(\cos^2 A + \sin^2 A) + b^2 - 2bc\cos A = c^2 + b^2 - 2bc\cos A$