Skid Pad Tests

- Stability - partly S.S.
- Responsiveness → Steering Sensitivity
- Controllability (Lane Change maneuver)
  - Often subjective

Stability Testing

1. Steady State
2. Transient
   - Sine w/ dwell — FMVSS 126
     - Steering following a sine curve, pauses on second hump.

Chapter 7 - Suspensions

- Main purpose: controls the wheels
  - Keeps the tires on the ground

ROLES

- Wheel Control (Keeps wheels on ground)
- Provide vertical compliance over
-
-

Basic Components: Springs & Dampers

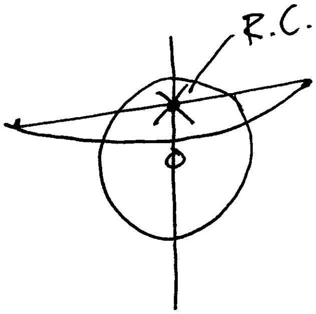
Solid Axles:
- Hotchkiss
- Four-link

Indepedent:
- MacPherson Strut
- SLA

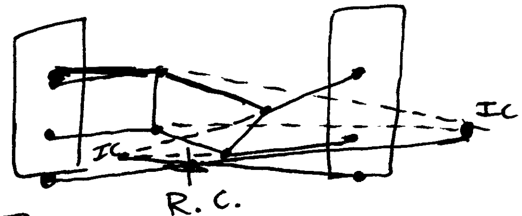
- Spring, wheel, tire, and Ride rates
  - Installation ratio ($K_w = K_s \times IS^2$, if IS is constant with stroke)
    - Same for dampers

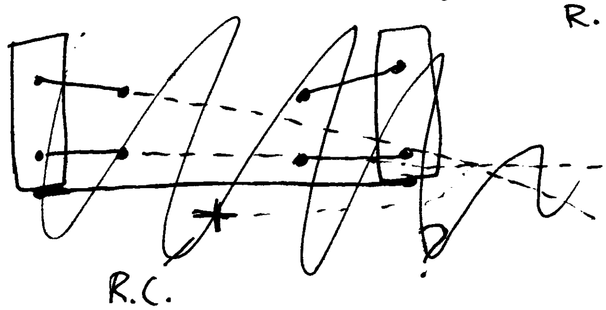
Roll Centers:

Solid Axle



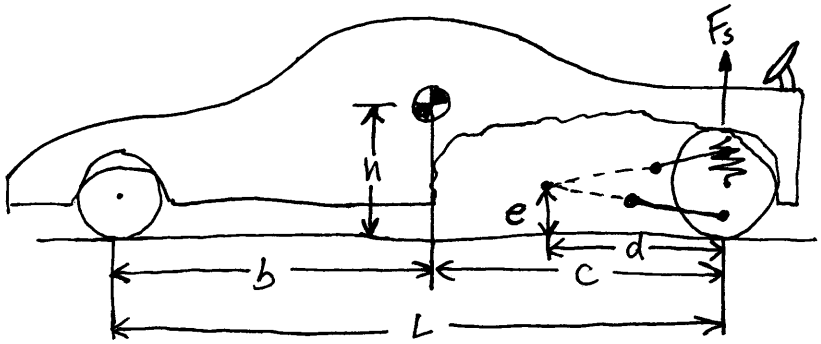
Independent

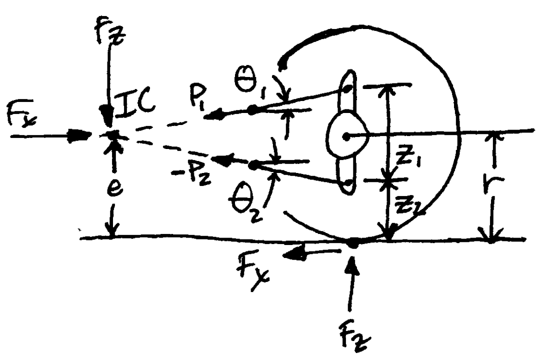




11/4/09

Anti-behavior





IC - instant center

$$P_1 = \frac{F_x z_2}{z_1 \cos\theta_1} \qquad \tan\theta_1 = \frac{z_2 + z_1 - e}{d}$$

$$P_2 = \frac{F_x(1 + z_2/z_1)}{\cos\theta_2} \qquad \tan\theta_2 = \frac{e - z_2}{d}$$

F_s = force thru rear spring

$F_z = \frac{e}{d} F_x$ = jacking force thru trailing arm

W_F = weight on front axle

$$F_s = \frac{Wb}{L} + F_x\left(\frac{h}{L} - \frac{e}{d}\right)$$

$$W_f = \frac{Wc}{L} - \frac{F_x h}{L}$$ — weight transfer

$$F_s = \frac{Wb}{L} + F_x\left(\frac{h}{L} - \frac{e}{d}\right)$$

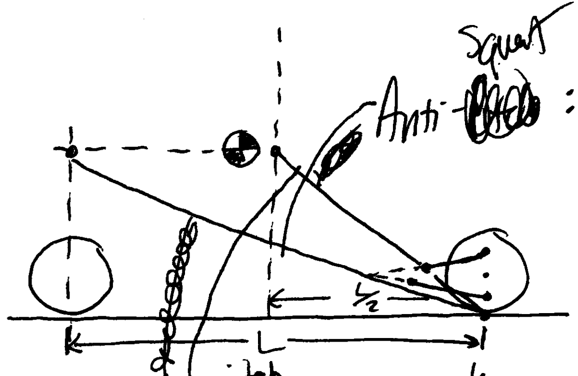
static wheel weights

- If $\left(\frac{e}{d} = \frac{h}{L}\right)$, the rear spring will not be loaded beyond static wheel weigh

= Anti-squat

• For anti-pitch, assuming front & rear springs have same stiffness

$$\boxed{\frac{e}{d} = \frac{h}{L/2}}$$ anti-pitch



Anti-Squat : $\frac{e}{d} = \frac{h}{L}$

Anti-pitch : $\frac{e}{d} = \frac{h}{(L/2)}$

11/11/09

Chap. 8 -

Steering

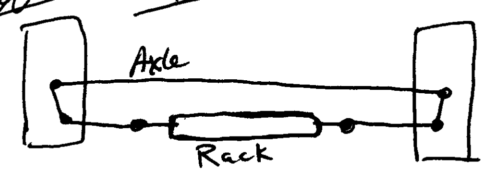
Steering Linkages

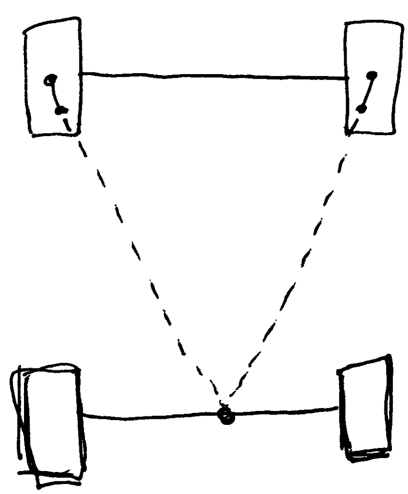
① Rack & Pinion

② Center Link / Steer Box

Kinematic Geometry

~~Backview~~ Top View





≈ 100% Ackerman Steering

• Degree to which Ackerman steering is achieved has little influence on high speed directional response, but does have influence on self-centering torque during low speed maneuvers

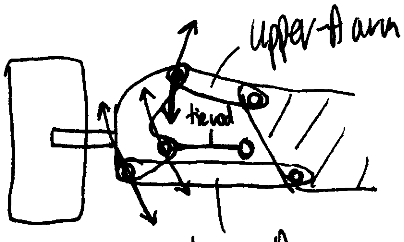
w/ Ackerman → steer-resisting torque grows consistently w/ steer angle

w/o Ackerman → torque initially grows, but then diminishes at larger angles.

steering geometry errors

- bump steer
- roll steer

- Fig. 8.9

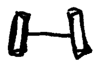


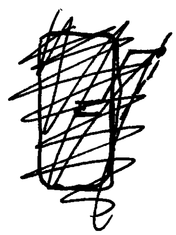
Roll Gradient

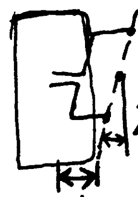
$$K_{roll\ steer} = \varepsilon \frac{d\phi}{d a_y}$$

Kingpin Inclination Angle $-\lambda$

Caster $-\nu$

$\rightarrow$ Toe in : 

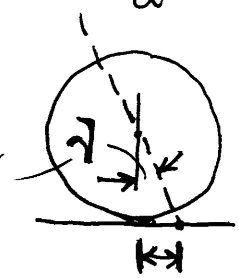




λ — kingpin inclination angle

used for: - steering stability (self-centering moment)
- reduces steering effort (scrub can be reduced)
- reduces tire wear

d — scrub length



caster

pneumatic trail offset

Casters :

pneumatic trail
mechanical trail
angle-caste
spindle-offset

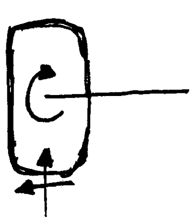
used for:
- directional stability
- jacking on one side can be used to an advantage
- avoids forward movement of pneumatic trail problem

HW#5

- Vertical loads on wheels
- Calculate front cornering force, back
- slip angle ? under/oversteer?

11/16/09

- Eq. 8.4 pg. 297 : FWD



4-wheel steering : low speed : steering out of phase
higher speeds : steer in phase

Dampers

Role: Suppress oscillation and control the motion of the sprung mass...

- Should not make up for a poor suspension.

Exam 2 (Chap. 6, 7, 8, Shock design, Suspension design)

11/19/09

Chap. 6

✓ – Ackerman Angle $\frac{L}{R}$

✓ – Slip Angle

✓ – Cornering Stiffness & Coeff.

✓ └ CC greatest in ~~heaviest~~ lightest? loads

✓ – Understeer grad. & steering sens.

✓ $\delta - \frac{L}{R} = \alpha_f - \alpha_r$

✓ $\alpha_r \cong \beta - \frac{cr}{V}$

✓ $\alpha_f \cong \beta + \frac{br}{V} - \delta$

✓ $a_y = \frac{V^2}{R} = V(r + \dot{\beta})$ u v r

✓ – pg. 226 table: understeer effects (list them)

✓ – under, over, neutral steer

✓ – critical speed

✓ – Eq. lateral load transfer

✓ └ Milliken

✓ └ 2 primary mechanisms: (roll center location, H)

✓ – Installation

$K_w = K_s (IR)^2$

– ranges of ride freqs of axles

Chap. 7

✓ – fcns of suspension

✓ – Identify designs (Hotz, 4-bar, Mc.-St, Swing)

✓ └ Swing (jacking, extreme camber change)

✓ – r.c. analysis

✓ – moving R.C.

✓ – anti-pitch, squat, dive (% conditions) problem?

Chap. 8

✓ – 2 basic steering systems

✓ – Steering gear errors (bump & rollsteer)

✓ – acker. steering geom.

✓ – steering ratio between rack & pin ~14 – 30s

✓ – Kingpin, caster, scrub

✓ └ self center, jacking concept

✓ – mech. trail (caster, spindle offset)

✓ – driveline torque steer

✓ – shock design lecture (fundamentals)

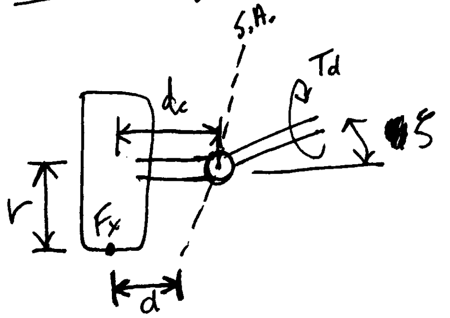
FORMAT:

Less short answer

~3 solution-type problems

– cheat sheet

Driveline Torque Steer pg. 297-8



λ = KP

ν = Caster

$$M_{S.A.} = F_x d \cos\nu \cos\lambda + T_d \sin(\lambda + \xi)$$

$$T_d = F_x r$$

$$M_{S.A.} = F_x \left[d \cos\nu \cos\lambda + r \sin(\lambda + \xi) \right]$$

– Assume λ, ν are small

$$M_{SA} = F_x \underbrace{\left[d + r \sin(\lambda + \xi) \right]}_{\text{effective moment arm}}$$

Imbalance in M_{SA} (right, left during turning) resists steer angle

→ steers out of turn

11/30/09

Hybrids

- 2 or more power sources, and an energy storage system
  - Usually electric motors, sometimes hydraulic pumps.

Series Hybrid or Parallel Hybrid:

Series – Electricity from a heat engine/generator APU powers drive motors and/or charges batteries

Parallel – Power from heat engine and drive motor are used independently or coupled together to power the vehicle and charge the batteries

Passenger Vehicles – Primarily parallel Hybrid

Goals of Parallel Hybridization:

1. Increase powertrain efficiency
   - Engine technology and downsizing
   - Transmission technology (AMT vs CVT vs Conventional Automatics)
   - Better control strategies (shift schedule, electric launch, engine-off at idle)

2. Decreasing powertrain loads
   - Aerodynamic reductions
   - rolling resistance
   - electric accessories
   - implementing regenerative braking

Desirable Features of Parallel Hybrids:

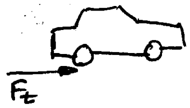
- Electric launch
- motor-assist and/or CVT operation
- Ability to use regenerative braking at all times
- Ability to power accessory loads at all times
- Charge sustaining operation (keeping the battery charged)

Basic Parallel Hybrid Transmission Architectures

- Electromechanical CVTs
  - Toyota THS (Prius)
  - Allison Ep
  - Other electromechanical clutch-types (Yamaguchi)

x	u	p - roll
y	v	q - pitch
z		r - yaw

Longitudinal Accel.



$$a_x = \left(\frac{F_t}{W}\right) g$$

1 mi = 5280 ft

$$HP = \frac{T(\text{lb-ft}) \times \dot{\theta}(\text{rad/s})}{550}$$

Engine, Transmission, Driveline:
$$\begin{cases} \omega_d = N_f\,\omega_w \\ \omega_e = N_t\,\omega_d = N_t N_f\,\omega_w \end{cases}$$

	F	D	I	OWC
1	X			X
2	X		X	
3	X	X		

Power

$P_{eng} = T \times \dot{\theta}$

$P(HP) = \frac{T \times RPM}{5252}$

$P_{veh} = F_t \times V$

Efficiency

$\eta = \frac{P_{veh}}{P_{eng}}$

Manual ≈ 92-97%

Auto ≈ 60-95%

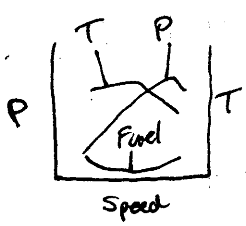
Power Loss Sources

- Gear Mesh
- Spin Losses
- Pumps
- Clutches
- Driveline (shafts, gears, tires/wheels)
- Tire Slip (hea

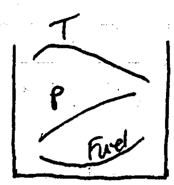
Newton's 2nd: $a_x = 550 \frac{g}{V} \frac{HP}{W}$ [Power Limited]

Engines

Engine Curves

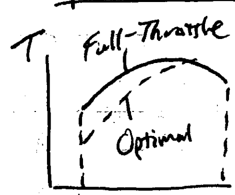


Gasoline

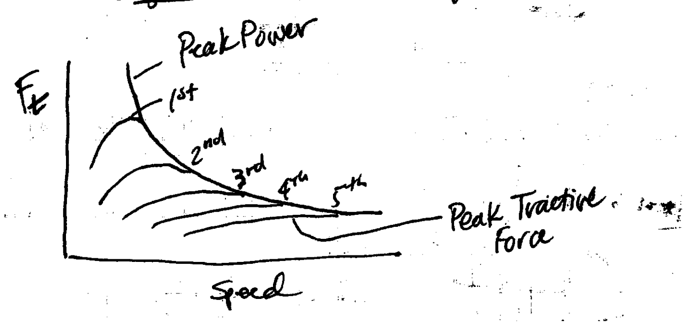


Diesal

Optimal Fuel Curve

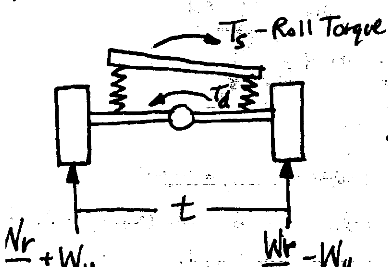


Engine Performance (Torque Curve)



Transmissions Needed for: Sufficient Torque, Driving at low speeds, extending vehicle's operational range (better use of eng powe

Traction-Limited Accel $F_x = \mu W$ μ - peak frictional coef., W - weight on drive wheel



- $W_y = (T_d - T_s)/t$ (Lateral Load Transfer)
- $T_d = F_x \frac{r}{N_f}$ — F_x - Tot. drive force from 2RW, r - Tire radius, N_f - F.D.R.
- $T_s = K_\phi \phi$
- $K_\phi = K_{\phi f} + K_{\phi r}$
- $\phi = \frac{T_d}{K_\phi}$
- K_ϕ = Roll Stiffness
- ϕ = Roll Angle
- $W_r = W\left(\frac{b}{L} + \frac{F_x}{mg}\frac{h}{L}\right)$
- $W_{rr} = \frac{W_r}{2} - W_y$
- $F_x = 2\mu W_{rr}$

Traction Limits:

RWD

$$F_{x\,max} = \frac{\mu \frac{Wb}{L}}{1 - \frac{h}{L}\mu + \frac{2\mu r}{N_f t}\frac{K_{\phi f}}{K_\phi}}$$ (solid rear axle w/ non-locking differential)

$$F_{x\,max} = \frac{\mu \frac{Wb}{L}}{1 - \frac{h}{L}\mu}$$ (solid rear axle w/ locking differential) (independent rear suspension) ←

FWD

$$F_{x\,max} = \frac{\mu \frac{Wc}{L}}{1 + \frac{h}{L}\mu + \frac{2\mu r}{N_f t}\frac{K_{\phi r}}{K_\phi}}$$ (solid front drive axle w/ non-locking diff.)

$$F_{x\,max} = \frac{\mu \frac{Wc}{L}}{1 + \frac{h}{L}\mu}$$ (solid front drive axle w/ locking diff.) (independent front drive axle)

Gear Steps

Geometric

$$i_{tot} = i_1/i_z = \frac{i_1}{i_2}\cdot\frac{i_2}{i_3}\cdots\frac{i_{z-1}}{i_z}$$
(overall g.r.)
$$= \phi_{12}\cdot\phi_{23}\cdots\phi_{(z-1)z}$$

$$\phi_g = \phi_{12} = \phi_{23} = \ldots$$

Progressive

$$\phi_{(z-1),z} = \frac{\phi_1}{\phi_f^{(z-2)}} = \frac{i_{z-1}}{i_z}$$

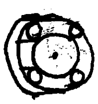
$$\phi_n = \phi_f^{(z-n-1)} \cdot \sqrt[z-1]{\frac{i_{tot}}{\phi_f^{(z-2)!}}}$$

$$\phi_{12} = \phi_1 = i_1/i_2$$

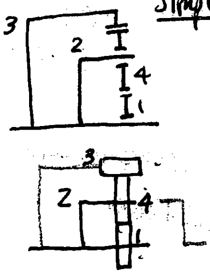
$$\phi_{23} = \phi_1/\phi_f = i_2/i_3$$

$$1.0 \le \phi_f \le 1.2$$

Planetary Gears



Simple



(Ravigneaux)

Mesh (i,j)	~~Transfer~~ Vertices (k)
$(4,3)$	(2)
$(4,1)$	(2)

- $\boxed{\omega_i - \omega_k = \pm N_{j,i}(\omega_j - \omega_k)}$

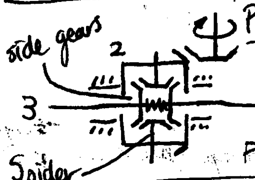
$$\omega_4 - \omega_2 = N_{3,4}(\omega_3 - \omega_2) \quad (1)$$
$$\omega_4 - \omega_2 = -N_{1,4}(\omega_1 - \omega_2) \quad (2)$$
$$\frac{(1)}{(2)} = \omega_1 - \omega_2 = N_{3,1}(\omega_2 - \omega_3)$$
$$\omega_1 = -N_{3,1}\omega_3$$

+ - external gear pair, − - internal gear pair

Simpson

- $\omega_1 - \omega_2 = N_{3,1}(\omega_2 - \omega_3)$
- $\omega_1 - \omega_3 = N_{4,1}(\omega_3 - \omega_4)$

Differentials (Open)



$$\omega_1 - \omega_2 = \omega_2 - \omega_3$$

2.75 : 1
4.11 : 1

$$\frac{\omega_1 + \omega_3}{2} = \omega_2$$

Pinion RPM is higher

Brakes

Passenger: Hydraulic (Disc, Drum)
Truck: Air

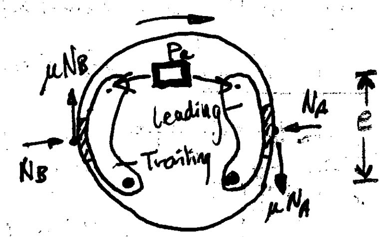
$$\Sigma F_x = ma_x = -\frac{W}{g}D_x = [-F_{xf} - F_{xr}] - D_A - W\sin\theta$$

$-F_{xf} - F_{xr}\}$ Brakes, RR, Bearing Friction, Driveline Drag

Drum Brakes - Brake Factor

$$\frac{F_A}{P_a} = \frac{\mu e}{(m - \mu n)} \qquad \frac{F_B}{P_a} = \frac{\mu e}{(m + \mu n)}$$

- F_A - Force leading shoe
- F_B - Force trailing shoe



Constant Decel.

- $D_x = \frac{F_{x,tot}}{m} = -\frac{dV}{dt}$
- $V_0 - V_f = \frac{F_{x,tot}}{m} t_s$
- $\frac{V_0^2 - V_f^2}{2} = \frac{F_{x,tot}}{m} X$
- $S.D. = \frac{V_0^2}{2D_x}$

Decel. w/ Wind Resis.

$$\Sigma F_x = F_b + CV^2$$

$$SD = \frac{m}{2C}\ln\left[\frac{F_b + CV_0^2}{F_b}\right]$$

Energy/Power

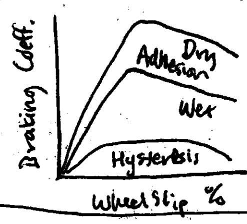
$$E = \frac{m}{2}(V_0^2 - V_f^2)$$

$$P = \frac{m}{2}\frac{V_0^2}{t_s}$$

Tire Slip

- $\text{Slip} = \frac{V - \omega r}{V}$ (%)

r - effective rolling rad.



Federal Requirements

135 - pass. cars (hydr.) < 7701
105 - other hyd. < 10,000
121 - air brakes

Road Loads

C_D (~0.25 − 1) passenger — Honda: 0.25, vette: 0.3

$$D_f = \frac{1}{2}\rho C_D A_f V^2$$

ρ/g

6-DOF, $\rho_{air} = .076\ \frac{lb}{ft^3} \Rightarrow .00236\ \frac{lb \cdot s^2}{ft^4}$

RR

- Tire temp
- Inflation Pressure
- Velocity dependancy
- Tire material/design
- Tire Slip
- Road conditions

RR Coeff

0.01 − 0.05

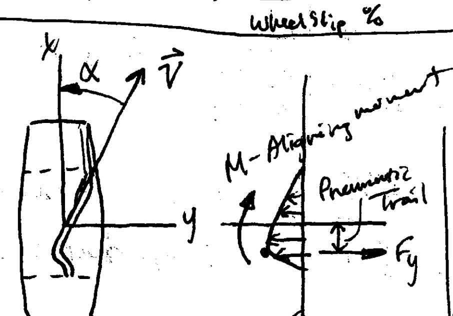
S.S. Cornering

$$\delta_0 \approx \frac{L}{R + t/2}$$

$$\delta_i \approx \frac{L}{R - t/2}$$

Ackerman Steering Angle - δ

$$\delta = \frac{L}{R} \text{ (rad)}$$
$$= 57.3\frac{L}{R} \text{ (deg)}$$



$$V_{char} = \sqrt{57.3\,Lg/K}$$
$$V_{crit} = \sqrt{-57.3\,Lg/K}$$

- K - Understeer Gradient

$$K = \frac{W_f}{C_{\alpha f}} - \frac{W_r}{C_{\alpha r}}$$

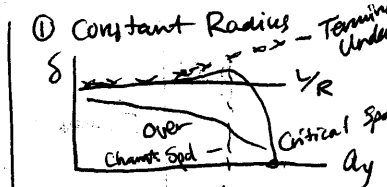
$$\boxed{\delta = \frac{L}{R} + Ka_y}$$

$$a_y = \frac{V^2}{R}$$

$$\cdot\ \beta = 57.3\frac{c}{R} - \alpha_r$$

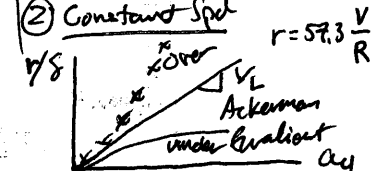
Skidpad Test

① Constant Radius



② Constant Spd

$$r = 57.3\frac{V}{R}$$



③ Constant Steer

Bike Model $\delta_0 \approx \delta_i$

$$\alpha_f = \left(\frac{W_f}{g}\right)\frac{V^2}{C_{\alpha f}R}$$

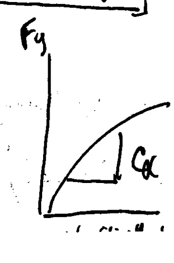
$$\alpha_r = \left(\frac{W_r}{g}\right)\frac{V^2}{C_{\alpha r}R}$$

C_α - cornering stiffness

Under: $\frac{W_f}{C_{\alpha f}} > \frac{W_r}{C_{\alpha r}}$

Over: $\frac{W_f}{C_{\alpha f}} < \frac{W_r}{C_{\alpha r}}$

$W_f - W_r$



Chap. 6 – Steady-State Cornering

Key assumptions: $R >> L$, $\delta_o \cong \delta_i$

Low-Spd (No Slip)

$\delta = \frac{L}{R}$

$\delta_o \cong \frac{L}{R + t/2}$

$\delta_i \cong \frac{L}{R - t/2}$

High-Spd Cornering

$F_y = C_\alpha \alpha$ — cornering stiffness

$CC_\alpha = \frac{C_\alpha}{F_z}$

– largest at light loads

– at 100% load, ≈ 0.2

Bicycle Model

$\delta = \frac{L}{R} + \alpha_f - \alpha_r$

$\alpha_f \cong \beta + \frac{br}{V} - \delta = \left(\frac{W_f}{g}\right)\frac{V^2}{C_{\alpha f} R}$

$\alpha_r \cong \beta - \frac{cr}{V} = \left(\frac{W_r}{g}\right)\frac{V^2}{C_{\alpha r} R}$

$\beta = \tan^{-1}\left(\frac{v}{u}\right)$ – sideslip angle

$F_{yr} = M\frac{b}{L}\frac{V^2}{R}$

$F_{yf} = M\frac{c}{L}\frac{V^2}{R}$

Critical Spd [During OVERSTEER] → leads to INSTABILITY

$V_{crit} = \sqrt{-57.3\, Lg/K}$

Lat. Load Transfer

2 primary mechanisms: ① R.C. location ② H

Understeer Gradient

$\delta = \frac{L}{R} + K a_y$, $\boxed{K\text{-undrstr grad } (\text{deg}/g)}$

$a_y = \frac{V^2}{R} = V(r + \dot{\beta})$ – transient

$= Vr$ – s.s.

1) Neutral

$\frac{W_f}{C_{\alpha f}} = \frac{W_r}{C_{\alpha r}}$ $\quad K = 0, \alpha_f = \alpha_r$

2) Under

$>$ $\quad K > 0, \alpha_f > \alpha_r$

3) Over

$<$ $\quad K < 0, \alpha_f < \alpha_r$

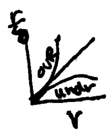
Lat. Accel. Gain

$\frac{a_y}{\delta} = \frac{\frac{V^2}{57.3 Lg}}{1 + \frac{KV^2}{57.3 Lg}}$ (deg/s)

Yaw Velocity Gain

$\frac{\vec{r}}{\delta} = \frac{V/L}{1 + \frac{KV^2}{57.3 Lg}}$

$\vec{r} = 57.3\frac{V}{R}$



Roll Moment Distribution

*Roll Stiffness: $K_\phi = 0.5 K_s s^2$ $\quad$ K_s – vertical rate of springs, s – lat. separation 'tween springs

$F_{zo} - F_{zi} = 2F_y\frac{h_{rc}}{t} + 2K_\phi\frac{\phi}{t}$

$= \underbrace{2\frac{W}{g}\frac{V^2}{R}\frac{h_{RC}}{t}}_{\text{Moment due to R.C. above ground}} + \underbrace{2\frac{W}{g}\frac{V^2}{R}\frac{H\cos\phi}{t} + 2\frac{WH}{t}\sin\phi}_{\text{Moment of C.G. above roll axis}}$

MILLIKEN:

$\frac{\Delta W_F}{a_y} = \frac{W}{t_f}\left[\frac{HK_f}{K_f + K_r} + \frac{c}{L}Z_{RC,f}\right]$

$\frac{\Delta W_R}{a_y} = \frac{W}{t_r}\left[\frac{HK_r}{K_f + K_r} + \frac{b}{L}Z_{RC,r}\right]$

K – roll stiffness

Z – R.C. height

Camber: γ $\quad$ $F_y = C_\alpha \alpha + C_\gamma \gamma$ → Camber Thrust

Understeer Effects: Lat. load transfer, camber change, roll steer, lat. force compliance steer, aligning torque, tire cornering stiff., steering system

Skid Pad Tests: stability (partly s.s.), responsiveness (steering sensitivity), controllability (lane change maneuver)

Chap. 7 – Suspensions

Functions: wheel control (keeps tires in contact), provide vertical compliance over rough terrain, React control forces at tires (accel, corn., braking), resists roll/pitch

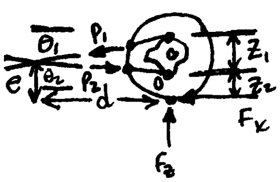
Types: Hotchkiss, 4-Link, MacPherson Strut, Swing Axle (issues w/ jacking, camber change), SLA (aka double A-arm, double wishbone)

Installation Ratio: $K_w = K_s \times IS^2$

Anti-Squat & Anti-Pitch:

$P_1 = \frac{F_x Z_2}{Z_1 \cos\theta_1}$ $\quad$ $\tan\theta_1 = \frac{Z_2 + Z_1 - e}{d}$

$P_2 = \frac{F_x(1 + Z_2/Z_1)}{\cos\theta_2}$ $\quad$ $\tan\theta_2 = \frac{e - Z_2}{d}$



Anti-Squat

$\frac{e}{d} = \frac{h}{L}$

Anti-Pitch

$\frac{e}{d} = \frac{2h}{L}$

Jacking: $F_z = \frac{e}{d}F_x$

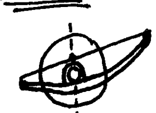
$W_f = \frac{Wc}{L} - \frac{F_x h}{L}$

$F_s = \frac{Wb}{L} + F_x\left(\frac{h}{L} - \frac{e}{d}\right)$

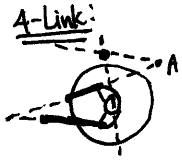
rear spring F

Roll Center Analysis:

Hotchkiss:

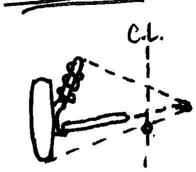


4-Link:



Independent:

MacPherson:



Anti-Dive:

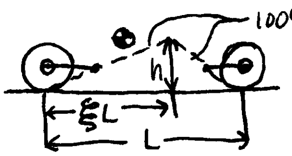
Front Suspension

$\frac{e_f}{d_f} = \tan\beta_f = \frac{h}{\xi L}$

Rear Suspension

$\frac{e_r}{d_r} = \tan\beta_r = \frac{h}{(1-\xi)L}$

ξ – Fraction of the brake force developed on the front axle

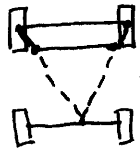


Max seldom exceeds 50%

*Understeer – Vehicle doesn't turn as much as intended

Chap. 8 - Steering

Steering Linkages:
① Rack & Pinion (Steering Ratio ~14-30)
② Center Link/Steer Box


→ ≈100% Ackerman

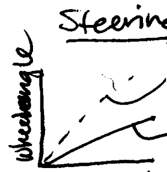
Steering Geometry Errors
① Bump Steer (both wheels rise together; suspension compressed) → toe out → understeer
② Roll Steer: $K_{roll\ steer} = \varepsilon \frac{d\phi}{da_y}$
toe in on out, toe out on inside wheel

Kingpin - λ
- steering stability (self-centering moment)
- reduces steering effort (scrub can be reduced)
- reduces tire wear

Trucks: 0-5 deg.
Cars: 10-15 deg.

Caster - ν
- pneumatic trail
- mech. trail (angle-caster, spindle-offset)
- directional stability
- jacking on one side can be used to advantage
- avoids forward movement of pneumatic trail problem

Scrub - d

Steering Ratio


passenger: (15-20):1
trucks: (20-36):1

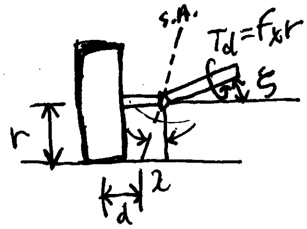
Driveline Torque Steer (contributes to understeer)
Net moment about S.A.:

$$M_{SA} = F_x\left[d\cos\nu\cos\lambda + r\sin(\lambda+\xi)\right]$$

small ν, λ:

$$M_{SA} = F_x\left[d + r\sin(\lambda+\xi)\right]$$

- imbalance in M_{SA} resists steer angle



Shocks
- Suppress oscillation & control motion of the sprung mass
  - roll
  - bounce
  - pitch
- pitch-bounce
- bounce-pitch
- Keeps tires on ground

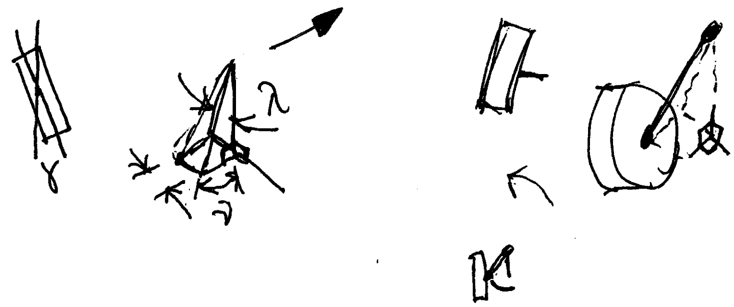
$w_n = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$, crit. damping: $C_{crit} = 2\sqrt{Km}$, $\xi = \frac{c}{C_{crit}}$ (~0.1-0.5)

pass. cars: 0.2-0.4
race cars: >1
off-road: 0.1-0.2

Telescopic designs:
① Mono-tube
- better heat dissipation
- more piston area for better sensitivity to small movement
- lighter

② Twin-tube
- more durable for off road

camber   kingpin

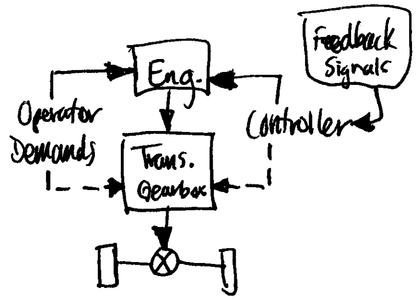


Chapter 2 Acceleration Performance

Power Limited Acceleration - acceleration is limited by engine power $a = \frac{g}{V}\frac{P}{W}$

$$a_x = 550\,\frac{g}{V}\frac{HP}{W} \quad ft/s^2$$

Powertrain Layout

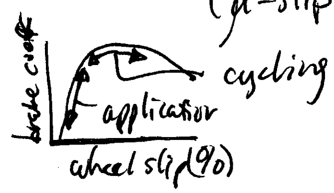


(Chap. 3)

$$t_s = \frac{V_0}{D_x}$$

ABS

· keeps you in peak braking coefficient (μ-slip) curve



(Chap. 4) Brake proportioning : used to balance outputs on front & rear axles based on peak frictional forces

Chap. 5 - Ride

U Joints :

Suspensions :

Ride Rate : $RR = \frac{K_s K_t}{K_s + K_t}$

Installation Ratio : $K_w = K_s \times IR^2$

Roll stiffness : $K_\phi = 0.5\, K_s\, s^2$

Roll gradient : $R_\phi = \frac{d\phi}{d a_y} = \frac{WH}{[K_{\phi f} + K_{\phi r} - WH]}$ $\left(\frac{deg}{g}\right)$

Ride Frequencies : (Indy : ~3 Hz, Pass : ~1 Hz)

Chapter 6

Roll Steer

ε - rollsteer coefficient $\left(\frac{\text{deg steer}}{\text{deg roll}}\right)$

$\therefore$ Understeer gradient from Roll Steer: $K_{rollsteer} = (\varepsilon_f - \varepsilon_r)\frac{\partial \phi}{\partial a_y} \left[\frac{\text{deg roll}}{g}\right]$ ϕ - roll angle

Understeer Tests

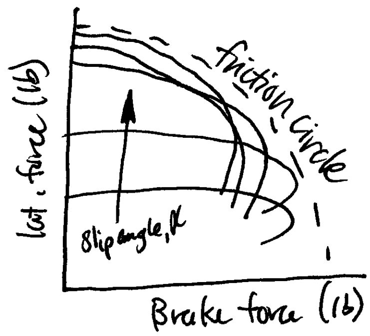
$\underbrace{\text{Constant radius, constant speed}}_{\text{reflect normal driving}}$, constangle steerangle, constant throttle

Chapter 10 - Tires

hysteresis = pressure

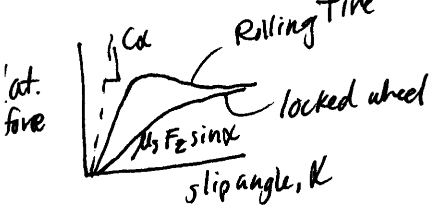
- steel belts give lat. stability

Traction Circle (Friction Circle):



↑ vertical load ↑ R.R.

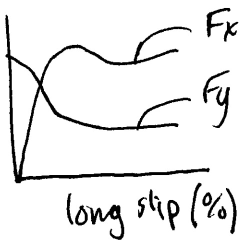
Cornering Stiffness



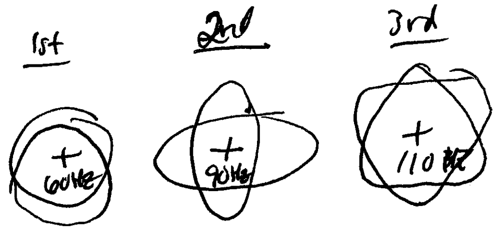
Slip (1st Cheat Sheet)

Long. Slip

long. slip ↑ lat. slip ↓



Vibrational Modes

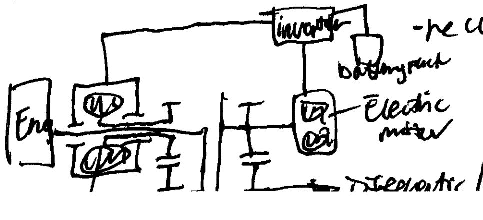


Hybrids

Classifications: Series, parallel

How they improve fuel economy:
- minimize the size of the heat engine (size is avg., not peak)
- allow engine to operate slower at a given power demand
- recuperate vehicle K.E. during braking

Prius:



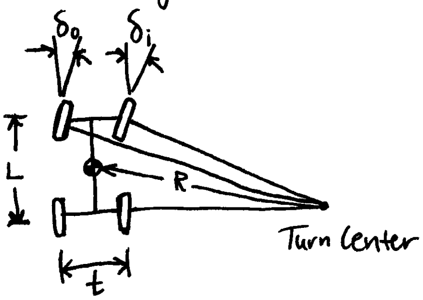
features:
- electric launch
- motor assist and/or CVT operation
- reg. braking
- power accessory loads
- ... operation

Chapter 6 - Steady State Cornering

LOW-SPEED TURNING (NO SLIP)

Ackerman Angle: $\delta = \frac{L}{R}$ (steering angle)

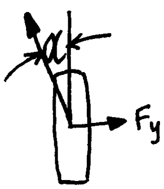
– Tires produce no lateral forces, no lateral slip



$$\delta_o \cong \frac{L}{R+\frac{t}{2}}$$

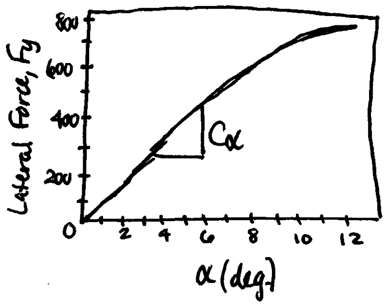
$$\delta_i \cong \frac{L}{R-\frac{t}{2}}$$

HIGH-SPEED CORNERING



slip angle: α

(aka cornering stiffness)
proportionality constant: C_α



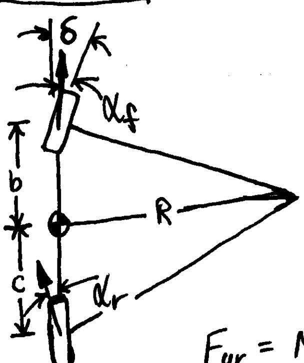
$$F_y = C_\alpha \alpha$$

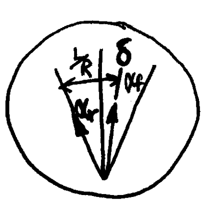
Cornering coefficient: $CC_\alpha = \frac{C_\alpha}{F_z}$

– usually largest at light loads, F_z

– at 100% load, $CC_\alpha \approx 0.2$

BIKE-MODEL





$$\delta = \frac{L}{R} + \alpha_f - \alpha_r \qquad \beta = \tan^{-1}\left(\frac{v}{u}\right)$$

$$\alpha_f \cong \beta + \frac{br}{V} - \delta = \left(\frac{W_f}{g}\right)\frac{V^2}{C_{\alpha f} R}$$

$$\alpha_r \cong \beta - \frac{cr}{V} = \left(\frac{W_r}{g}\right)\frac{V^2}{C_{\alpha r} R}$$

β – sideslip angle (angle between long. axis and local direction of trave

$$F_{yr} = M\frac{b}{L}\frac{V^2}{R}$$

$$F_{yf} = M\frac{c}{L}\frac{V^2}{R}$$

UNDERSTEER GRADIENT

$\delta = \frac{L}{R} + K a_y$, K - understeer gradient $\left(\frac{deg}{g}\right)$, $a_y = \frac{V^2}{R} = V(r + \dot{\beta})$ – transient

$a_y = Vr$ – s.s. ($\dot{\beta} = 0$)

1) Neutral Steer

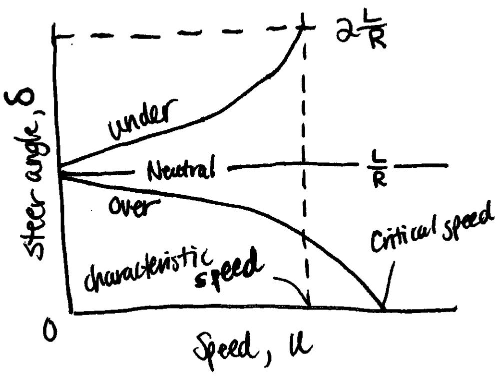
$\frac{W_f}{C_{\alpha f}} = \frac{W_r}{C_{\alpha r}}$ $\quad K=0,\ \alpha_f = \alpha_r$

2) Understeer

$\frac{W_f}{C_{\alpha f}} > \frac{W_r}{C_{\alpha r}}$ $\quad K>0,\ \alpha_f > \alpha_r$

3) ~~[illegible]~~ Oversteer

$\frac{W_f}{C_{\alpha f}} < \frac{W_r}{C_{\alpha r}}$ $\quad K<0,\ \alpha_f < \alpha_r$



CRITICAL SPEED

$$V_{crit} = \sqrt{-57.3\, Lg/K}$$

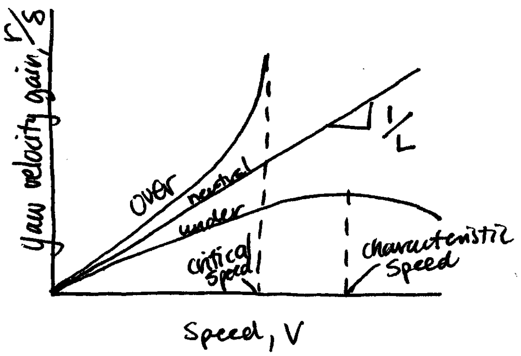
LATERAL ACCELERATION GAIN

$$\frac{a_y}{\delta} = \frac{\dfrac{V^2}{57.3\,Lg}}{1+\dfrac{KV^2}{57.3\,Lg}} \quad \left(\text{deg}/\text{sec}\right)$$

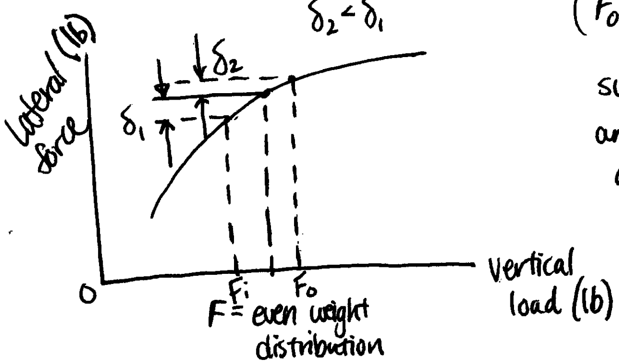
YAW VELOCITY GAIN

$$\vec{r} = 57.3\,\frac{V}{R}$$

$$\frac{\vec{r}}{\delta} = \frac{V/L}{1+\dfrac{KV^2}{57.3\,Lg}}$$



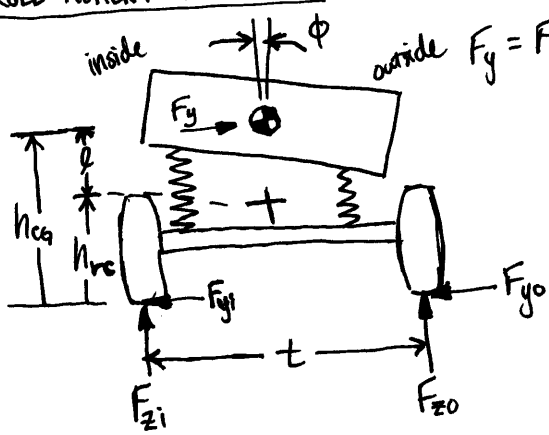
SUSPENSION EFFECTS ON CORNERING



$$\left(F_o + F_i\right) < 2F$$

sum of vertical forces on inside & outside are always less than sum at even distribution (2F)

ROLL MOMENT DISTRIBUTION



$F_y = F_{yi} + F_{yo}$

Roll Stiffness: $K_\phi = 0.5\,K_s\,S^2$

K_s – vertical rate of each of the left & right springs

S – lateral separation between springs

$$F_{zo} - F_{zi} = 2F_y\frac{h_r}{t} + 2K_\phi\frac{\phi}{t}$$

$$= \underbrace{2\frac{W}{g}\frac{V^2}{R}\frac{h_r}{t}}_{\text{Moment due to R.C. above ground}} + \underbrace{2\frac{W}{g}\frac{V^2}{R}\frac{l\cos\phi}{t} + 2\frac{Wl}{t}\sin\phi}_{\text{Moment of C.G. above roll axis}}$$

Lateral Load Transfer

2 primary mechanisms:

1) roll center location
2) H

Milliken:

$$\frac{\Delta W_F}{a_y} = \frac{W}{t_f}\left[\frac{H K_f}{K_f + K_r} + \frac{c}{L} Z_{rf}\right]$$

$$\frac{\Delta W_R}{a_y} = \frac{W}{t_r}\left[\frac{H K_r}{K_f + K_r} + \frac{b}{L} Z_{rr}\right]$$

K – roll stiffness

H – height of C.G. above Roll Axis

Z – roll center height

UNDERSTEER EFFECTS

- Lateral Load Transfer
- Camber Change
- Roll Steer
- Lateral Force Compliance Steer
- Aligning Torque
- Tire Cornering Stiffness
- Steering system

SKID PAD TESTS

Stability - partly S.S.
Responsiveness - steering sensitivity
Controllability (lane change maneuver)

Chapter 7 - Suspensions

FUNCTIONS OF SUSPENSIONS

- Wheel Control (keeps tires in contact)
- Provide Vertical compliance over rough terrain
- React control forces at tires (accel, cornering, braking)
- Resist body roll/pitch

COMMON TYPES

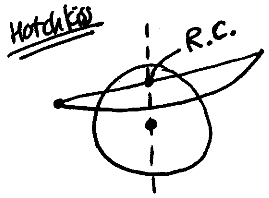
$\left.\begin{array}{l}\text{- Hotchkiss} \\ \text{- 4-Link}\end{array}\right\}$ Solid Axles

$\left.\begin{array}{l}\text{- MacPherson Strut} \\ \text{- Swing Axle (issues w/ jacking \& camber change)} \\ \text{- SLA (Short long arm, Double A-arm, Double wishbone)}\end{array}\right\}$ independe

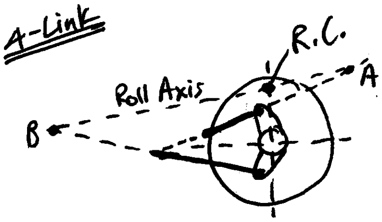
ROLL CENTER ANALYSIS

Solid Axles:

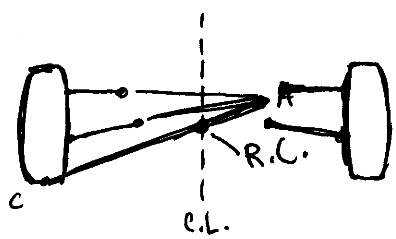
Hotchkiss



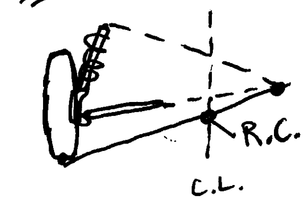
4-Link

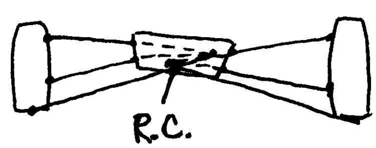


Independent Axles:



MacPherson

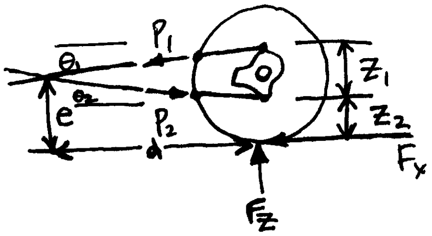




INSTALLATION

Installation Ratio: $K_w = K_s \times IR^2$

ANTI-SQUAT AND ANTI-PITCH SUSPENSION GEOMETRY



$$P_1 = \frac{F_x Z_2}{Z_1 \cos\theta_1} \qquad \tan\theta_1 = \frac{Z_2 + Z_1 - e}{d}$$

$$P_2 = \frac{F_x(1 + Z_2/Z_1)}{\cos\theta_2} \qquad \tan\theta_2 = \frac{e - Z_2}{d}$$

Jacking Force:

$$F_Z = \frac{e}{d} F_x$$

$$W_f = \frac{Wc}{L} - \frac{F_x h}{L}$$

$$F_s = \frac{Wb}{L} + F_x\left(\frac{h}{L} - \frac{e}{d}\right)$$

Anti-Squat

$$\frac{e}{d} = \frac{h}{L}$$

Anti-Pitch

$$\frac{e}{d} = 2\frac{h}{L}$$

ANTI-DIVE SUSPENSION GEOMETRY

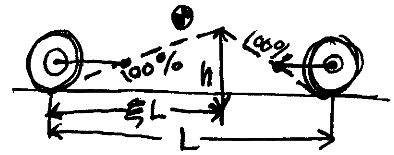
Front Suspension

$$\frac{e_f}{d_f} = \tan\beta_f = -\frac{h}{\xi L}$$

Rear Suspension

$$\frac{e_r}{d_r} = \tan\beta_r = \frac{h}{(1-\xi)L}$$

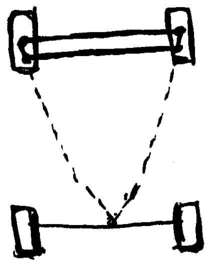
ξ - Fraction of the brake force developed on the front axle



- Max anti-dive seldom exceeds 50%

Chapter 8 - The Steering System

Steering Linkages:
1) Rack & Pinion
2) Center Link / Steer Box



$\approx 100\%$ Ackerman Steering

Rack & Pinion

Steering ratio ~14-30s

STEERING GEOMETRY ERRORS

Bump Steer

Roll Steer

$$K_{rollsteer} = \varepsilon \frac{d\phi}{da_y}$$

KINGPIN, CASTER, SCRUB

KINGPIN - λ
Trucks: ~0-5 deg
Pass.: ~10-15 deg

- used for steering stability (self-centering moment)
- reduces steering effort (scrub can be reduced)
- reduces tire wear

CASTER - ν

- pneumatic trail
- mech. trail (angle-caster, spindle-offset)

- directional stability
- jacking on one side can be used to an advantage
- avoids forward movement of pneumatic trail problem

SCRUB - d

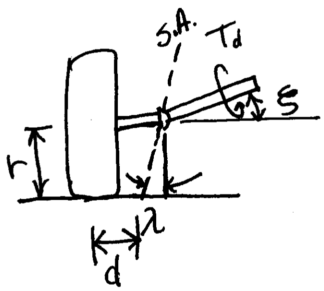
DRIVELINE TORQUE STEER

Net Moment about Steer Axis:

$$M_{SA} = F_x\left[d\cos\nu\cos\lambda + r\sin(\lambda+\xi)\right]$$

small λ, ν:

$$M_{SA} = F_x\left[d + r\sin(\lambda+\xi)\right]$$



- imbalance in M_{SA} resists steer angle

SHOCKS

- suppress oscillation & control motion of the sprung mass
  - roll
  - bounce
  - pitch
- Keep tires on ground

- pitchy-bounce
- bouncy-pitch

$$\omega_n = \frac{1}{2\pi}\sqrt{\frac{K}{m}}, \quad \text{crit. damping: } C_{crit} = 2\sqrt{Km}, \quad \xi = \frac{c}{C_{crit}} \quad (\sim 0.1 - 0.5)$$

passenger cars: (0.2-0.4)
race cars: (>1)
off-road: (0.1-0.2)