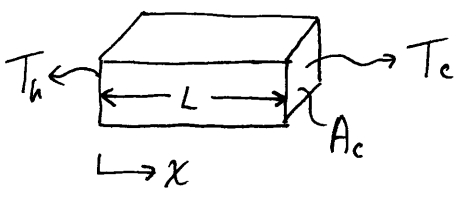


# ENME332 - Heat Transfer

8/31/09

3 modes of heat transfer: conduction, convection, radiation

conduction - molecular vibrations/electronic transfer



$$\dot{q} \propto \frac{\Delta T}{L} A_c$$

$$\Delta T = T_h - T_c$$

so we expect that

$$\dot{q} = k A_c \frac{\Delta T}{L} \quad \text{Fourier's Law}$$

nomenclature:

$$\begin{matrix} \dot{q} [W] & \dot{q}' [W/m] \\ \dot{q}'' [W/m^2] & \dot{q}''' [W/m^3] \end{matrix} \quad (\text{book})$$

$$\dot{q} = k A_c \frac{dT}{dx} \quad \text{as } L \rightarrow 0$$

$$\text{units: } [\dot{q}] = W = J/s$$

$$[A_c] = m^2$$

$$\left[ \frac{dT}{dx} \right] = \frac{^\circ C}{m} = \frac{K}{m}$$

$$[k] = \left[ \frac{\dot{q}}{A_c \frac{dT}{dx}} \right] = \frac{W}{m \cdot K} = \frac{W}{m \cdot K}$$

Overdot = "per unit time"  
prime = "per unit length"

$$\dot{q} [W] \equiv [J/s] \equiv \dot{q}$$

$$\dot{q}''' [W/m^3] \equiv \dot{q}'''$$

$$\dot{q}' [W/m] \equiv \dot{q}'$$

$$\dot{q}'' [W/m^2] \equiv \dot{q}''$$

professor's                      book's

property values in book:

thermal conductivity of air:

air properties	
T (K)	$K \times 10^3 \frac{W}{m \cdot K}$
100	~
200	~
300	26.3

$$K_{air} = 26.3 \times 10^{-3} \frac{W}{m \cdot K}$$

$$K_{air} \neq 26.3 \times 10^3 \frac{W}{m \cdot K}$$

gases:

-  $K_{gases}$  is independent of pressure

$$K_{air} = 0.026 \frac{W}{m \cdot K}$$

$$K_{helium} = 0.175 \frac{W}{m \cdot K}$$

$$K_{hydrogen} = 0.175 \frac{W}{m \cdot K}$$

$$K_{CO_2} = 0.015 \frac{W}{m \cdot K}$$

liquids:

-  $k_{\text{liquids}}$  depends on temperature, but can increase/decrease depending on fluid & temperature

$$k_{\text{water}} = 0.556 \frac{\text{W}}{\text{m}\cdot\text{K}}$$

$$\text{oil} = 0.147 \frac{\text{W}}{\text{m}\cdot\text{K}}$$

$$\text{mercury} = 8.21 \frac{\text{W}}{\text{m}\cdot\text{K}}$$

solids:

- lattice vibration    electronic transport

$$k_{\text{glass}}: 1.5 \frac{\text{W}}{\text{m}\cdot\text{K}}$$

$$k_{\text{stainless steel}}: 15 \frac{\text{W}}{\text{m}\cdot\text{K}}$$

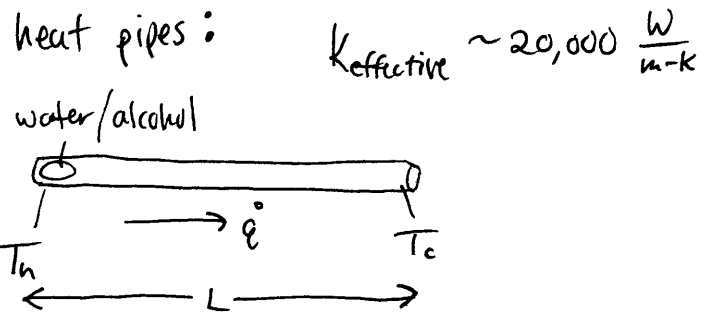
$$k_{\text{silicon}}: 120 \frac{\text{W}}{\text{m}\cdot\text{K}}$$

$$k_{\text{sapphire}}: 70 \frac{\text{W}}{\text{m}\cdot\text{K}}$$

$$k_{\text{aluminum}}: 200 \frac{\text{W}}{\text{m}\cdot\text{K}}$$

$$k_{\text{copper}}: 400 \frac{\text{W}}{\text{m}\cdot\text{K}}$$

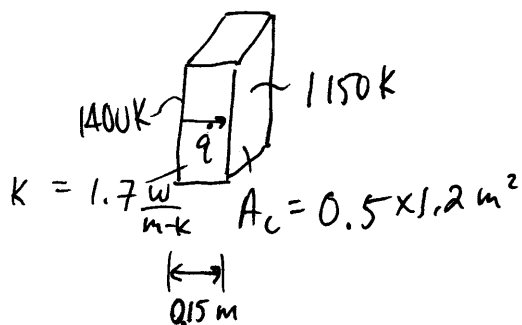
$$k_{\text{diamond}}: 2000 \frac{\text{W}}{\text{m}\cdot\text{K}}$$



ex) walls of a furnace made of brick ( $k = 1.7 \frac{\text{W}}{\text{m}\cdot\text{K}}$ ) wall is 0.15 m thick

$$T_{\text{inner}} = 1400 \text{ K}, \quad T_{\text{outer}} = 1150 \text{ K}$$

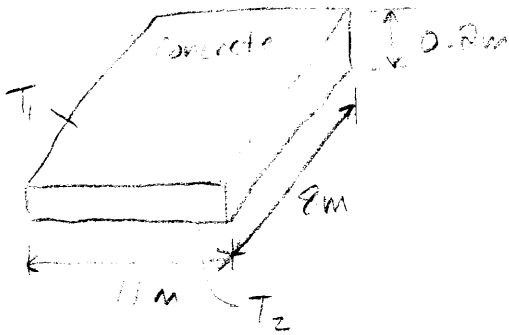
what is heat loss over a  $0.5 \times 1.2 \text{ m}^2$  area?



$$\begin{aligned} \dot{q} &= kA \frac{dT}{dx} \\ &= \left(1.7 \frac{\text{W}}{\text{m}\cdot\text{K}}\right) (0.5 \text{ m} \times 1.2 \text{ m}) \left(\frac{-1400 \text{ K} + 1150 \text{ K}}{0.15 \text{ m}}\right) \\ &= -1700 \text{ W} \end{aligned}$$

9/1/09

1.3)



$$k = \frac{1}{\frac{1}{k_1} + \frac{1}{k_2}}$$

$$T_1 = 17^\circ\text{C}$$

$$T_2 = 10^\circ\text{C}$$

$$\eta_f = 0.9 \text{ efficiency}$$

$$\text{cost} = \$0.01/\text{MJ}$$

$$\dot{q} = -k A_c \frac{\Delta T}{L}$$

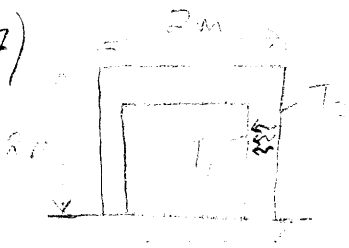
$$= -\left(\frac{1}{\frac{1}{k_1} + \frac{1}{k_2}}\right) (11\text{m} \times 8\text{m}) \frac{(17 - 10)^\circ\text{C}}{0.2\text{m}} = 4,318 \text{ W}$$

$$\eta_f = \frac{\dot{q}_0}{\dot{q}_1} = \frac{\dot{q}_0}{\dot{q}_1} \rightarrow \dot{q}_0 = \frac{4,318 \text{ W}}{0.9} = 4,799 \text{ W}$$

$$J_{\text{day}} = (\text{cost}) \rightarrow (4,799 \text{ J/s}) (86,400 \text{ s}) = 414 \text{ MJ in 1 day}$$

$$(\$0.01/\text{MJ})(414 \text{ MJ}) = \boxed{\$4.14 \text{ per day}}$$

1.7)



$$T_1 = -10^\circ\text{C}$$

$$T_2 = 35^\circ\text{C}$$

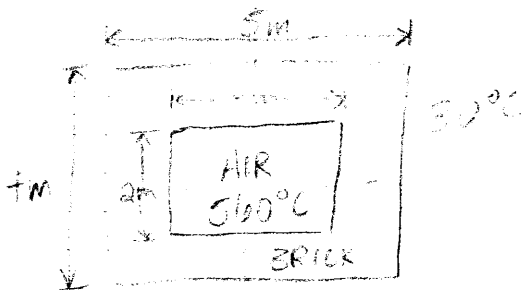
$$k = 0.08 \frac{\text{W}}{\text{m}\cdot\text{K}}$$

$$\dot{q} = 500 \text{ W}$$

$$\dot{q} = -k A_c \frac{\Delta T}{L}$$

$$500 \text{ W} = \left(-0.08 \frac{\text{W}}{\text{m}\cdot\text{K}}\right) (5 \times 2 \text{ m}^2) \frac{-10 - 35^\circ\text{C}}{t}$$

$$t = \boxed{0.054 \text{ m}}$$



$$\dot{Q} = -hA_c \Delta T$$

heat transfer coefficient  
in direction

$$k_{\text{air}} = 10$$

$$k_{\text{brick}} = 5$$

1. elp

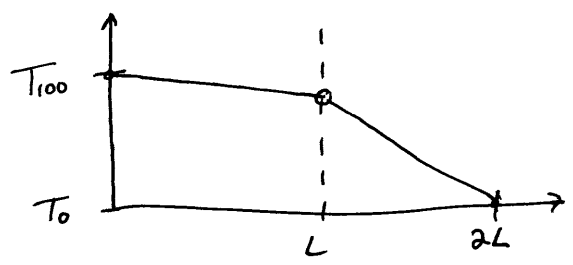
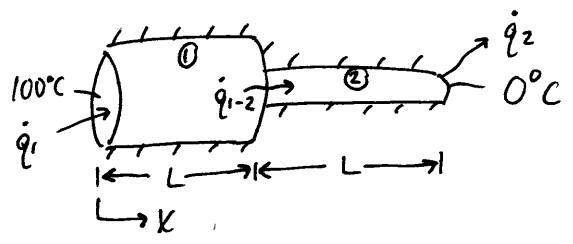
2. find out the

3. draw

4. solve

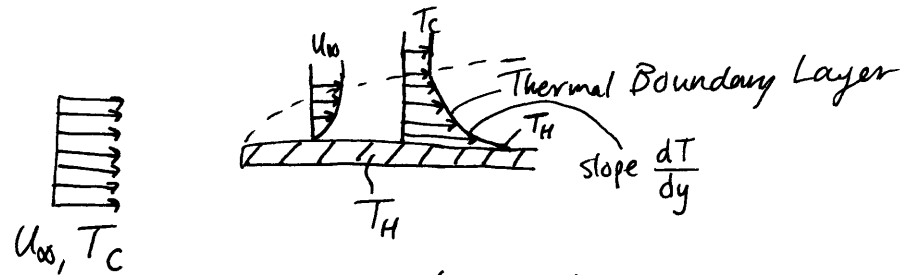
example for Heat Conduction:

$$\dot{q} = -kA \frac{dT}{dx}$$



Steady state: all heat flowing must flow out

Convection - Heat transfer by bulk fluid motion

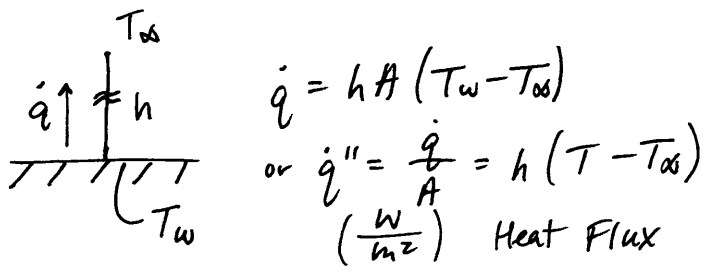


$$\dot{q} = -kA \frac{dT}{dy}$$

\*  $\dot{q} = h A_s (T_w - T_{\infty})$

surface area available for convection

heat transfer coefficient  $[h] = \left[ \frac{-\dot{q}}{A_s (T_w - T_{\infty})} \right] = \frac{W}{m^2 \cdot K} \equiv \frac{W}{m^2 \cdot ^\circ C}$



Types of Convection:

Forced Convection - Fluid forced over a surface (wind tunnel) (Boiling & condensation)

Free Convection - (Natural convection)  $\Rightarrow$



Free Convection in air :  $h \cong 6-10 \frac{W}{m^2-K}$

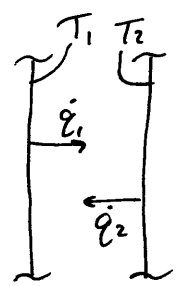
" " " water :  $h \cong 1000 \frac{W}{m^2-K}$

Forced Convection in air :  $h \cong 10-100 \frac{W}{m^2-K}$

" " " Water :  $h \sim 3000 \frac{W}{m^2-K}$

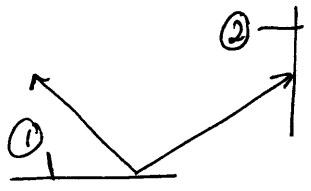
Pool Boiling in water :  $2500-35,000 \frac{W}{m^2-K}$

Suppose we had two infinite plates facing each other :



The net radiation exchange between 1 & 2 is given by :

What if surfaces don't see each other completely?



use a view factor to account for this

$$\dot{q}_{1-2}'' = \sigma F_y (T_1^4 - T_2^4)$$

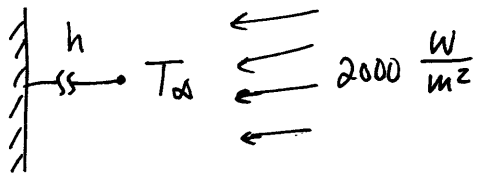
|  
view factor

Non-black body behavior is accounted for by an "emissivity"

$$\dot{q}_{1-2}'' = \sum \sigma F_y (T_1^4 - T_2^4)$$

$0 < \epsilon < 1$        $\epsilon = 1 \Rightarrow$  black body

ex) coating on a plate is cured by exposure to IR radiation that provides uniform illumination @  $2000 \frac{W}{m^2}$ . The surface is also exposed to an air flow @  $20^\circ C$ . The surroundings are at  $30^\circ C$ . If the surface absorbs 80% of incoming radiation and has  $\epsilon = 0.50$ , what is the temperature of the surface if  $h = 15 \frac{W}{m^2 \cdot K}$ ?



\* Assume S.S.  $\rightarrow$  heat in = heat out

$\hookrightarrow$  Energy absorbed = Energy lost

$$2000 \frac{W}{m^2} \times 0.80 = h (T_w - T_{\infty}) + \epsilon \sigma (T_w^4 - T_{\infty}^4)$$

$$2000 \frac{W}{m^2} \times 0.80 = 15 \frac{W}{m^2 \cdot K} (T_w - 20^\circ C) + (0.50) \sigma (T_w^4 - 30^\circ C^4)$$

$+273 K$ 
 $+273 K$

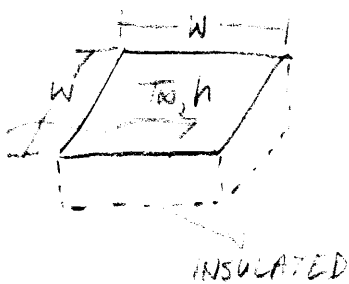
<must be Kelvin>
<must be Kelvin>

$$\boxed{T_w = 377 K}$$

9/8/09

1.18)  $W = 5\text{mm}$   $h = 200 \frac{\text{W}}{\text{m}^2\cdot\text{K}}$  air coolant, maximum allowable power?

$T_{\infty} = 15^{\circ}\text{C}$   
 $T_{\text{max}} = 85^{\circ}\text{C}$



$$\dot{q} = -h A_c \Delta T$$

$$= \left( -200 \frac{\text{W}}{\text{m}^2\cdot\text{K}} \right) (0.005\text{m} \times 0.005\text{m})^2 (15^{\circ}\text{C} - 85^{\circ}\text{C})$$

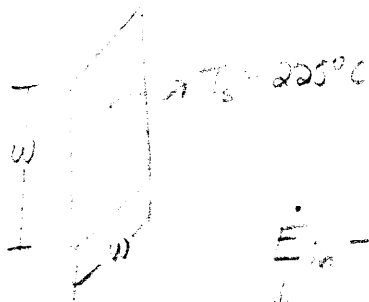
$$= \boxed{0.25 \text{ W}}$$

$h = 3000 \frac{\text{W}}{\text{m}^2\cdot\text{K}}$  dielectric liquid

$$\dot{q} = -\left( 3000 \frac{\text{W}}{\text{m}^2\cdot\text{K}} \right) (0.005\text{m} \times 0.005\text{m})^2 (15^{\circ}\text{C} - 85^{\circ}\text{C})$$

$$= \boxed{5.25 \text{ W}}$$

1.22) Thin vertical plate suspended in still air



$\frac{dT}{dt} = -0.022 \text{ K/s}$   
 Ambient air =  $25^{\circ}\text{C}$

$w = 0.3 \text{ m}$   
 $m = 3.75 \text{ kg}$   
 $C_p = 2770 \frac{\text{J}}{\text{kg}\cdot\text{K}}$

$$\dot{E}_{\text{in}} - \dot{E}_{\text{out}} = \dot{E}_{\text{change}}$$

$$0 - h A (T_s - T_{\infty}) = m C_p \frac{dT}{dt} \longrightarrow \dot{E}_{\text{in}} = h A \frac{dT}{dt}$$

$$h = \frac{-m C_p \frac{dT}{dt}}{A (T_s - T_{\infty})} = \frac{(-3.75 \text{ kg}) (2770 \frac{\text{J}}{\text{kg}\cdot\text{K}}) (-0.022 \text{ K/s})}{(0.3 \text{ m} \times 0.3 \text{ m}) (225^{\circ}\text{C} - 25^{\circ}\text{C})}$$

$$= 12.7$$



1.22) Uninsulated steam pipe

surrounding walls & air at  $25^{\circ}\text{C}$

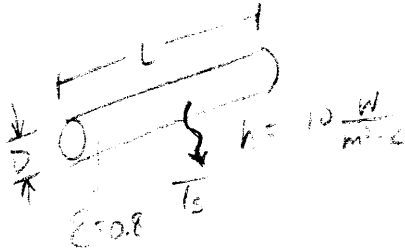
$$L = 25\text{ m}$$

$$D = 100\text{ mm}$$

pipe surface temperature of  $150^{\circ}\text{C}$

$$h = 10 \frac{\text{W}}{\text{m}^2\text{-K}}, \quad \epsilon = 0.8$$

a) rate of heat loss?



$$\dot{Q}_{\text{conv}} = -hA\Delta T$$

$$= \left(-10 \frac{\text{W}}{\text{m}^2\text{-K}}\right) \left(\pi \cdot 0.1\text{ m} \cdot 25\text{ m}\right) (150^{\circ}\text{C} - 25^{\circ}\text{C})$$

$$= -9817 \text{ W}$$

new factor

$$\dot{Q}_{\text{rad}} = \epsilon\sigma(T_s^4 - T_a^4)F_s$$

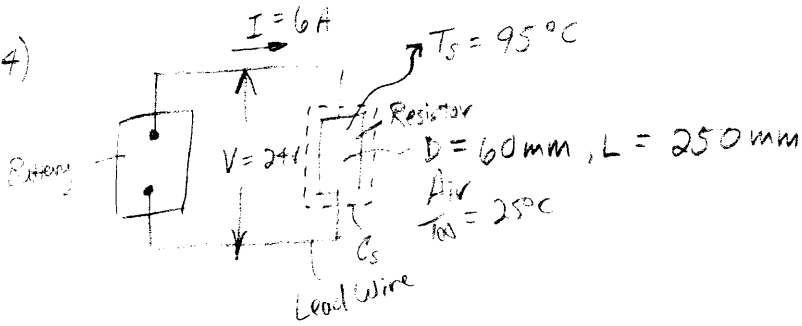
$$= (0.8) (5.67 \times 10^{-8} \text{ W/m}^2\text{-K}^4) (150^4 - 25^4) (\pi \cdot 0.1\text{ m} \cdot 25\text{ m})$$

=

b)  $\eta_f = 0.90$ ,  $C_g = \$0.01/\text{MJ}$  annual cost of heat loss?

$$E = 5.7 \times 10^{11} \text{ J} = 18450 \times 3600 \times 24 \times 365$$

1.34)



a) Find  $\dot{E}_{in}$ ,  $\dot{E}_g$ ,  $\dot{E}_{out}$ ,  $\dot{E}_{st}$

$$\dot{E}_{st} \equiv \frac{d\dot{E}_{st}}{dt} = \dot{E}_{in} - \dot{E}_{out} + \dot{E}_g$$

$$\begin{aligned} \dot{E}_{in} &= 0 \\ \dot{E}_{out} &= 144 \text{ W} \\ \dot{E}_{st} &= 0 \\ \dot{E}_g &= IV = 144 \text{ W} \end{aligned}$$

} Control surface of Resistor

b) Volumetric heat generation rate?

$$\begin{aligned} &= \\ &= 2.09 \times 10^9 \frac{\text{W}}{\text{m}^3} \end{aligned}$$

c)  $h$ ?

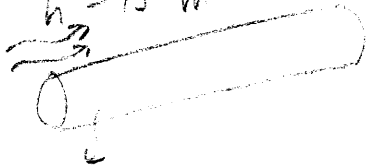
$$\begin{aligned} \dot{E}_{out} &= hA_{tot} (T_s - T_{air}) \\ 144 \text{ W} &= h(\pi DL + (\frac{\pi D^2}{4}) \times 2) (95^\circ \text{C} - 25^\circ \text{C}) \end{aligned}$$

$$h = 38.98 \frac{\text{W}}{\text{m}^2 \cdot \text{K}}$$

1.64)

$$T_{\infty} = 25^{\circ}\text{C}$$

$$h = 15 \frac{\text{W}}{\text{m}^2 \cdot \text{K}}$$



$$T_s = 200^{\circ}\text{C}$$

$$D = 70 \text{ mm}$$

$$\epsilon = 0.8$$

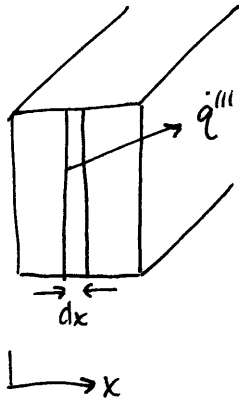
$$a) \quad G = \sigma T_{\text{sur}}^4$$

$$\frac{E_{\text{net}}}{L} = E_{\text{rad}} - \alpha G$$

$$= \pi D \left[ \epsilon \sigma (T_s^4) - \alpha \sigma T_{\text{sur}}^4 \right]$$

9/9/09

Heat of Diffusion Eq.



$$\dot{q}_{in} + \dot{q}_{gen} - \dot{q}_{out} = \dot{q}_{st}$$

$$\dot{q}_{in} = \dot{q}_x = -kA \frac{\partial T}{\partial x}$$

$$\dot{q}_{gen} = \dot{q}''' A dx \quad \dot{q}''' = \left[ \frac{W}{m^3} \right]$$

$$\dot{q}_{out} = -kA \frac{\partial T}{\partial x} \Big|_{x+dx}$$

$$= -kA \left[ \frac{\partial T}{\partial x} + \frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) dx \right]$$

$$\dot{q}_{st} = mC \frac{\partial T}{\partial t}$$

$$-kA \frac{\partial T}{\partial x} + \dot{q}''' A dx - kA \frac{\partial T}{\partial x} + A \frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) = \dot{q}_{st} = \rho C_p \frac{\partial T}{\partial t} A dx$$

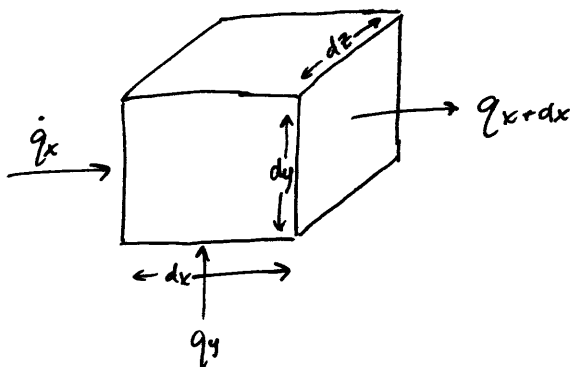
$$\dot{q}''' A dx + A \frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) = \rho C_p \frac{\partial T}{\partial t} A dx$$

1-D

$$\boxed{\frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) + \dot{q}''' = \rho C_p \frac{\partial T}{\partial t}}$$

- Heat of Diffusion Eq.

3-D



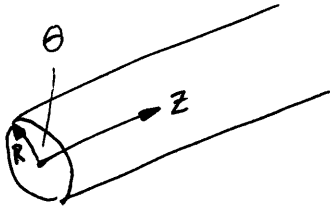
$$\frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( k \frac{\partial T}{\partial z} \right) + \dot{q}''' = \rho C_p \frac{\partial T}{\partial t}$$

$k = \text{constant}$  (divide thru by  $k$ )

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}'''}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial \tau}$$

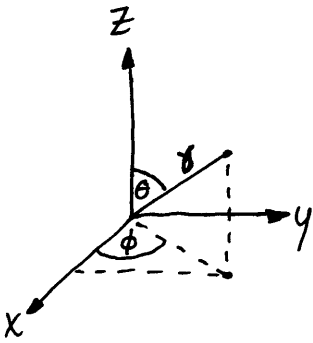
$$\alpha = \frac{k}{\rho c_p} = \text{thermal diff}^2$$

### Cylindrical Coordinates



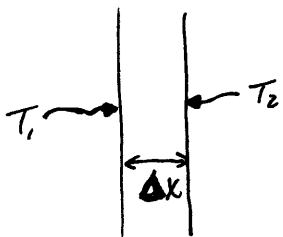
$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}'''}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial \tau}$$

### Spherical Coordinates



$$\frac{1}{r} \frac{d^2}{dr^2} (rT) + \frac{1}{r^2 \sin \theta} \frac{d}{d\theta} \left( \sin \theta \frac{dT}{d\theta} \right) + \frac{1}{r^2 \sin^2 \theta} \left( \frac{d^2 T}{d\phi^2} \right) + \frac{\dot{q}'''}{k} = \frac{1}{\alpha} \frac{dT}{d\tau}$$

### Steady State 1-D Convection



$$\frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) = 0$$

$$k \frac{\partial T}{\partial x} = D$$

$$\frac{\partial T}{\partial x} = A \leftarrow \frac{D}{k} = A$$

integrate  $\rightarrow T = Ax + B$   
 Boundary Conditions:  
 $T = T_1$  @  $x = 0$   
 $T = T_2$  @  $x = \Delta x$

$$T_1 = A \cdot 0 + B = B$$

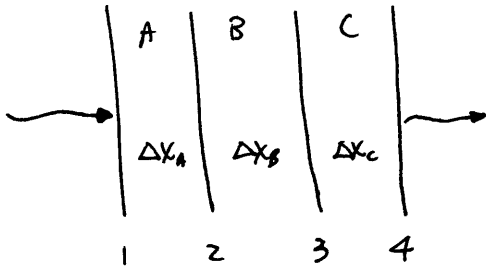
$$T_2 = A \Delta x + B$$

$$T = \frac{(T_2 - T_1)x}{\Delta x} + T_1$$

$k = k_0 (1 + \beta T)$   
 - not a fcn. of  $x, y, z$   
 - a function of  $T$

$$q = -\frac{k_0 A}{\Delta x} \left[ (T_2 - T_1) + \frac{\beta}{2} (T_2^2 - T_1^2) \right]$$

Composite Wall (S.S.)



$$q = -K_A A_A \frac{(T_2 - T_1)}{\Delta x_A}$$

$$A_A = A_B = A_C$$

$$= -K_B A_B \frac{(T_3 - T_2)}{\Delta x_B}$$

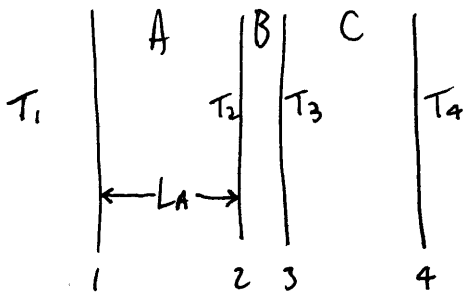
$$= -K_C A_C \frac{(T_4 - T_3)}{\Delta x_C}$$

$$(T_1 - T_2) + (T_2 - T_3) + (T_3 - T_4) = \frac{q}{A} \left[ \frac{\Delta x_A}{K_A} + \frac{\Delta x_B}{K_B} + \frac{\Delta x_C}{K_C} \right]$$

$$q = - \frac{K_{eff} A (T_4 - T_1)}{\Delta x_A + \Delta x_B + \Delta x_C}$$

$$K_{eff} = \frac{\Delta x_A + \Delta x_B + \Delta x_C}{\frac{\Delta x_A}{K_A} + \frac{\Delta x_B}{K_B} + \frac{\Delta x_C}{K_C}}$$

9/14/09



$$\dot{q}_A = \dot{q}_B = \dot{q}_C$$

$$-K_A \frac{dT}{dx} \Big|_A = -K_B \frac{dT}{dx} \Big|_B = -K_C \frac{dT}{dx} \Big|_C$$

# Electrical Resistance Analogy

Electricity

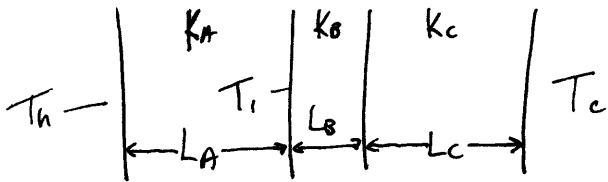
$$\begin{array}{c} I \\ \Delta V \\ R \end{array}$$

Heat Transfer

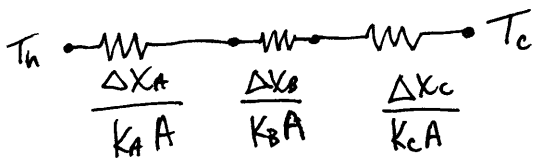
$$\begin{array}{c} \dot{q} \\ \Delta T \\ R_{th} \end{array}$$

$$\dot{q} = -kA \frac{\Delta T}{\Delta x} \equiv I = \frac{V}{R}$$

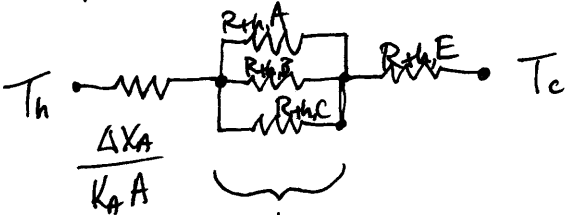
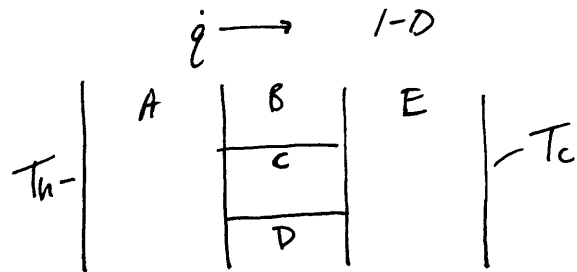
$$= \frac{\Delta T}{R_{th}} \rightarrow \boxed{R_{th} = \frac{\Delta x}{kA}}$$



$$\dot{q} = \frac{T_h - T_c}{\sum R_{th}}$$



$$T_i: \dot{q} = \frac{T_h - T_i}{\frac{\Delta x_A}{k_A A}}$$

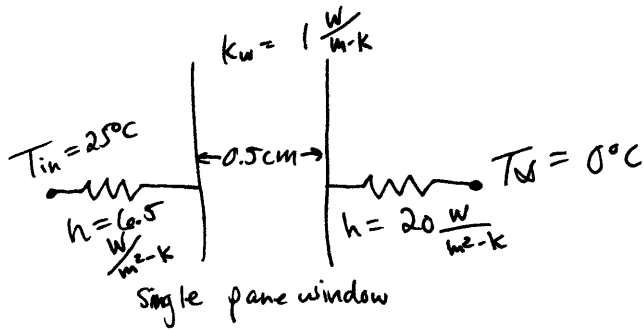


$$\frac{1}{R_{eq}} = \frac{1}{R_{th,A}} + \frac{1}{R_{th,B}} + \frac{1}{R_{th,C}}$$

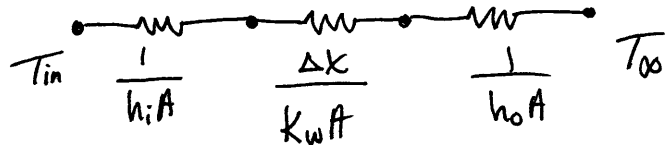
## Convection Resistances

$$\dot{q} = hA(T_w - T_\infty)$$

$$= \frac{T_w - T_\infty}{\frac{1}{hA}} = \frac{T_w - T_\infty}{R_{th}} \quad \rightarrow \quad R_{th} = \frac{1}{hA}$$



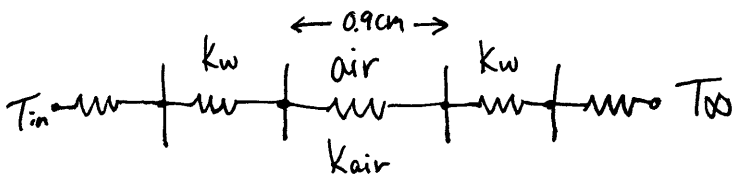
What is the heat flow per unit area?



$$\dot{q} = \frac{T_i - T_\infty}{\frac{1}{h_i A} + \frac{\Delta x}{k_w A} + \frac{1}{h_o A}} \Rightarrow \frac{\dot{q}}{A} = \dot{q}'' = \frac{T_i - T_\infty}{\frac{1}{h_i} + \frac{\Delta x}{k_w} + \frac{1}{h_o}}$$

$$= \frac{25^\circ\text{C} - 0^\circ\text{C}}{\frac{1}{6.5 \frac{\text{W}}{\text{m}^2\cdot\text{K}}} + \frac{0.005\text{m}}{1 \frac{\text{W}}{\text{m}\cdot\text{K}}} + \frac{1}{20 \frac{\text{W}}{\text{m}^2\cdot\text{K}}}} = 120 \frac{\text{W}}{\text{m}^2}$$

double pane window:



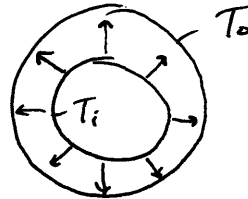
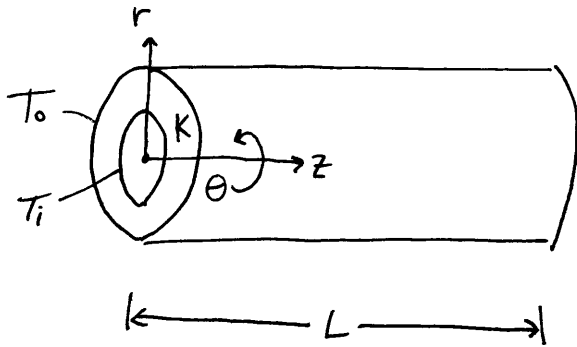
$$\dot{q}'' = \frac{\dot{q}}{A} = \frac{T_i - T_\infty}{\frac{1}{h_i} + \frac{\Delta x_w}{k_w} + \frac{\Delta x_{air}}{k_{air}} + \frac{\Delta x_w}{k_w} + \frac{1}{h_o}}$$

$$\dot{q}'' = 44 \frac{\text{W}}{\text{m}^2}$$



## Cylindrical Systems

- Consider a long hollow cylinder of length  $L$ , inner radius  $r_i$ , outer radius  $r_o$



$$\dot{q} = -kA \frac{dT}{dr}$$

$$A(r) = 2\pi rL$$

$$\dot{q} = \frac{-k2\pi rL}{r} \frac{T_o - T_i}{\ln(r_o/r_i)}$$

gov. eq:  $\frac{d^2T}{dr^2} + \frac{1}{r} \frac{dT}{dr} + \frac{1}{r^2} \frac{d^2T}{d\theta^2} + \frac{d^2T}{dz^2} + \frac{\dot{q}'''}{k} = \frac{1}{\alpha} \frac{dT}{dt}$

only dependent on radius:

$$\frac{d^2T}{dr^2} + \frac{1}{r} \frac{dT}{dr} = 0$$

$$\frac{d}{dr} \left( \frac{dT}{dr} \right) + \frac{1}{r} \frac{dT}{dr} = 0$$

$$\int \frac{d\left(\frac{dT}{dr}\right)}{\frac{dT}{dr}} = \int -\frac{1}{r} dr$$

$$\rightarrow \ln\left(\frac{dT}{dr}\right) = -\ln(r) + A \rightarrow \frac{dT}{dr} = C \frac{1}{r}$$

$$T = C \ln(r) + D$$

use B.C.:  $T = T_i$  @  $r = r_i$ ,  $T = T_o$  @  $r = r_o$

$$\boxed{\frac{T - T_i}{T_o - T_i} = \frac{\ln(r/r_i)}{\ln(r_o/r_i)}}$$

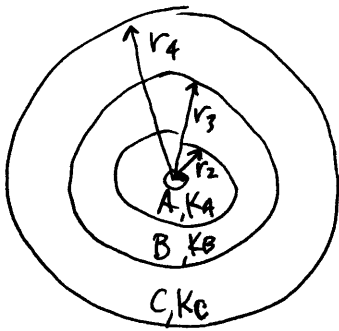
Temp. distribution  
in cylinder

$$\dot{q} = \frac{T_i - T_o}{\frac{\ln(r_o/r_i)}{2\pi kL}} = \frac{T_i - T_o}{R_{th}}$$

$$\boxed{R_{th} = \frac{\ln(r_o/r_i)}{2\pi kL}}$$

9/16/09

Multi-Layered Cylinders

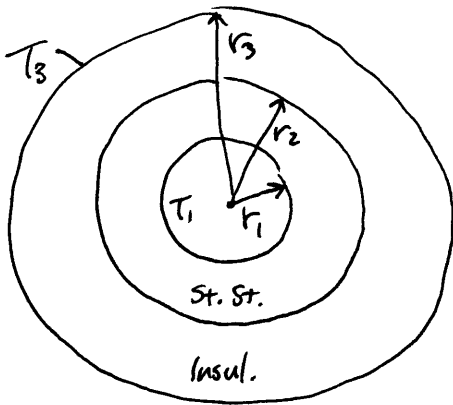


$r_3, T_3$   
 $r_4, T_4$   
 $r_2, T_2$   
 $r_1, T_1$

$$R_{th} = \frac{\ln\left(\frac{r_o}{r_i}\right)}{2\pi KL}$$

$$\dot{q} = \frac{\Delta T_{overall}}{R_{th,A} + R_{th,B} + R_{th,C}}$$

$$= \frac{(T_1 - T_4) 2\pi L}{\frac{\ln(r_2/r_1)}{K_A} + \frac{\ln(r_3/r_2)}{K_B} + \frac{\ln(r_4/r_3)}{K_C}}$$



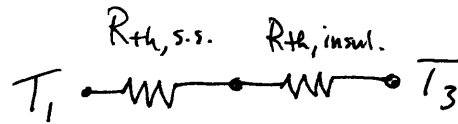
$T_1 = 600^\circ C$

$T_3 = 100^\circ C$

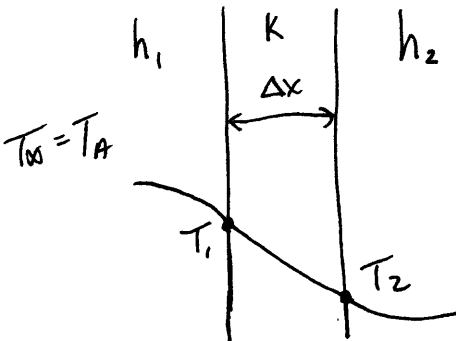
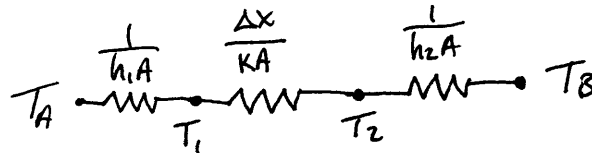
$r_1 = 0.01 m$

$r_2 = 0.02 m$

$r_3 = 0.05 m$



Overall H.T. Coeff.



$$\dot{q} = \frac{\Delta T}{\sum R_{th}} = \frac{T_A - T_B}{\frac{1}{h_1 A} + \frac{\Delta x}{KA} + \frac{1}{h_2 A}}$$

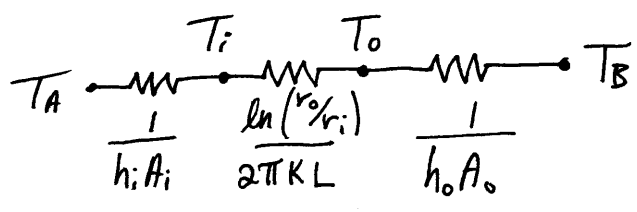
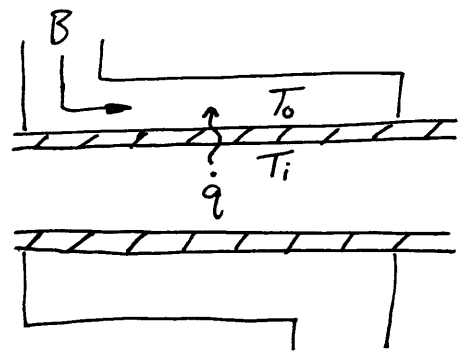
$$\dot{q} = UA \Delta T_{overall}$$

$$U = \frac{1}{\frac{1}{h_1} + \frac{\Delta x}{K} + \frac{1}{h_2}}$$

U - overall H.T. coeff.

# Parallel Flow Heat Exchanger

Hollow Tube:



$$A_i = 2\pi r_i L$$

$$A_o = 2\pi r_o L$$

$$\dot{q} = \frac{\Delta T_{overall}}{\sum R_{th}} = \frac{T_A - T_B}{\frac{1}{h_i A_i} + \frac{\ln(r_o/r_i)}{2\pi K L} + \frac{1}{h_o A_o}}$$

$$U_i = \frac{\dot{q}}{A_i (T_A - T_B)}$$

$$U_i = \frac{1}{\frac{1}{h_i} + \frac{A_i \ln(r_o/r_i)}{2\pi K L} + \frac{A_i}{h_o A_o}}$$

$$U_o = \frac{\dot{q}}{A_o (T_A - T_B)}$$

$$U_o = \frac{1}{\frac{A_o}{A_i h_i} + \frac{A_o \ln(r_o/r_i)}{2\pi K L} + \frac{1}{h_o}}$$

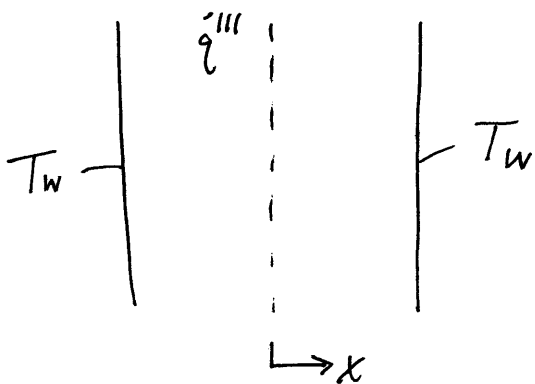
## R Value for Insulation

$$R = \frac{\Delta T}{\dot{q}A} = \frac{\Delta T}{\frac{KA\Delta T}{\Delta x A}} = \frac{\Delta x}{K} \quad \begin{matrix} \uparrow \\ \downarrow \end{matrix} \quad R \uparrow$$

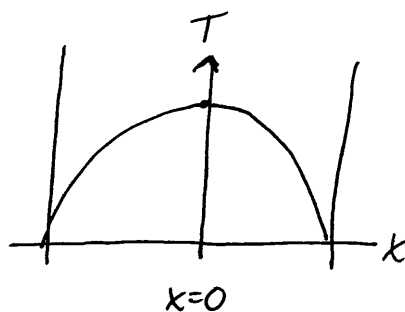
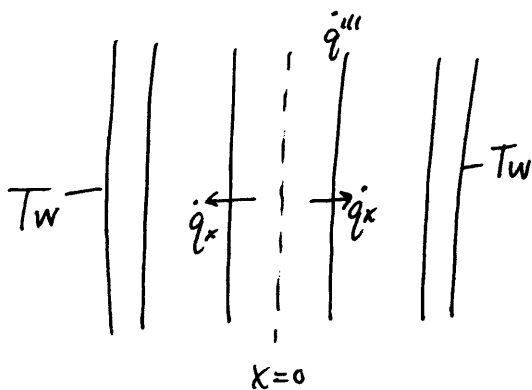
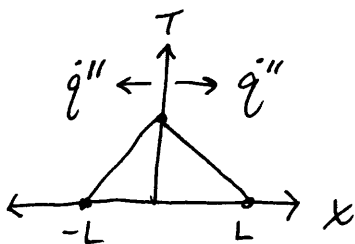
- 6" Fiberglass  $R = 3 \frac{W}{m^2K}$

9/21/09

## Internal Heat Generation (Cannot use resistance network analogy)



steady state  
 $\dot{q}''' = \text{const.}$



Derive Temperature distribution in slab:

$$\frac{d}{dx} \left( k \frac{\partial T}{\partial x} \right) + \frac{d}{dy} \left( k \frac{\partial T}{\partial y} \right) + \frac{d}{dz} \left( k \frac{\partial T}{\partial z} \right) + \dot{q}''' = \rho c_p \frac{dT}{dt}$$

5.5.

$$\frac{d}{dx} \left( k \frac{\partial T}{\partial x} \right) = -\dot{q}''' \quad k = \text{const.}$$

$$\frac{d}{dx} \left( \frac{\partial T}{\partial x} \right) = -\frac{\dot{q}'''}{k} \rightarrow d \left( \frac{dT}{dx} \right) = -\frac{\dot{q}'''}{k} dx$$

$$\frac{dT}{dx} = -\frac{\dot{q}'''}{k} x + A$$

$$T = -\frac{\dot{q}'''}{2k} x^2 + Ax + B$$

$$\text{B.C. : } T = T_w \text{ @ } x = L, T = T_w \text{ @ } x = -L$$

Solve for A, B to obtain

$$T - T_w = -\frac{\dot{q}'''}{2k} (x^2 - L^2)$$

$$T_{\text{max}} \text{ occurs @ } x=0 : T_{\text{max}} = T_w + \frac{\dot{q}''' L^2}{2k}$$

$$\text{or } T_{\text{max}} - T_w = +\frac{\dot{q}''' L^2}{2k}$$

Non-dimensional temperature profile:

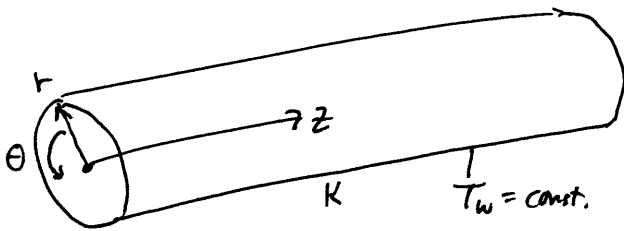
$$\frac{T - T_w}{T_{\text{max}} - T_w} = 1 - \frac{x^2}{L^2}$$



# Cylindrical Systems

- solid cylinder, radius  $R$ ,  $\dot{q}''' = \text{const.}$ , S.S.

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}'''}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$



symmetric axis, S.S.,  $\dot{q}''' = \text{const.}$

$$\frac{d^2 T}{dr^2} + \frac{1}{r} \frac{dT}{dr} = -\frac{\dot{q}'''}{k}$$

B.C.:  $T = T_w @ r = R, \frac{\partial T}{\partial r} = 0 @ r = 0$

Alternate B.C.: All energy generated in cylinder must be transferred by conduction at the wall.

$$\dot{q}''' \times V = \dot{q}''' \times \pi R^2 L = k A_s \left. \frac{dT}{dr} \right|_{r=R}$$

REWRITE

$$\dot{q}''' \pi R^2 L = k 2\pi R L \left. \frac{dT}{dr} \right|_{r=R}$$

$$\dot{q}''' R = 2k \left. \frac{dT}{dr} \right|_{r=R} \leftarrow \text{Alt. B.C.}$$

~~$\frac{d}{dr} \left( r \frac{dT}{dr} \right) = \frac{d^2 T}{dr^2} + \frac{1}{r} \frac{dT}{dr}$~~

$$r \frac{d^2 T}{dr^2} + \frac{dT}{dr} = -\frac{\dot{q}''' r}{k}$$

$$\frac{d}{dr} \left( r \frac{dT}{dr} \right) = -\frac{\dot{q}''' r}{k} \xrightarrow{\text{INTEGRATE}} r \frac{dT}{dr} = -\frac{\dot{q}''' r^2}{2k} + C$$

Apply B.C.:  $\frac{dT}{dr} = 0 @ r = 0 \rightarrow C = 0$

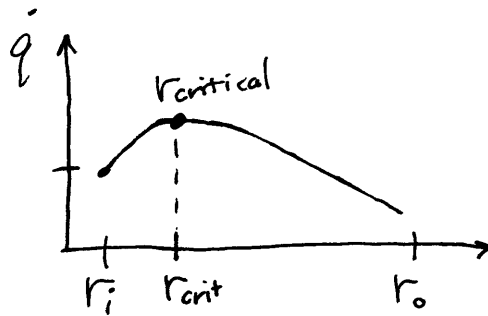
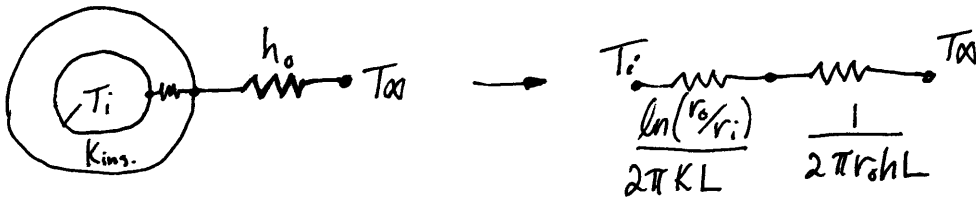
$$\frac{dT}{dr} = -\frac{\dot{q}''' r}{2k} \rightarrow T = -\frac{\dot{q}''' r^2}{4k} + C_2$$

$$\text{B.C.: } T = T_w \text{ @ } r = R \rightarrow C_2 = T_w + \frac{\dot{q}''' R^2}{4k}$$

$$T - T_w = \frac{\dot{q}'''}{4k} (R^2 - r^2) \quad \text{@ } r = 0 \quad T_0 = \frac{\dot{q}''' R^2}{4k} + T_w \quad \left( \begin{array}{l} \text{max Temp.} \\ \text{in cylinder} \end{array} \right)$$

discussion 9/22/09

## Critical Radius of Insulation



$$\dot{q} = \frac{T_i - T_\infty}{\frac{\ln(r_o/r_i)}{2\pi k L} + \frac{1}{2\pi L h r_o}}$$

To find  $r_{crit}$ :  $\frac{d\dot{q}}{dr_o} = 0$

$$r_{crit} = \frac{k}{h}$$

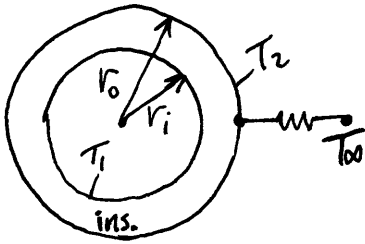
$$= \frac{(T_i - T_\infty) 2\pi L k h}{h \ln(r_o/r_i) + \frac{k}{r_o}} = \frac{(T_i - T_\infty) 2\pi L k h}{h(\ln r_o - \ln r_i) + \frac{k}{r_o}} = \frac{(T_i - T_\infty) 2\pi L k h r_o}{h \ln(r_o) r_o}$$

$$\frac{d\dot{q}}{dr_o} = \frac{d}{dr_o} \left( (T_i - T_\infty) \left( \frac{\ln(r_o/r_i)}{2\pi k L} + \frac{1}{2\pi L h r_o} \right)^{-1} \right)$$

$$= (T_\infty - T_i) \left( \right)$$



ex)



$$T_i = 200^\circ\text{C} \quad h = 3 \frac{\text{W}}{\text{m}^2\cdot\text{K}} \quad r_i = 2.5 \text{ cm}$$

$$T_\infty = 20^\circ\text{C} \quad k_{\text{ins}} = 0.17 \frac{\text{W}}{\text{m}\cdot\text{K}}$$

- what is  $\dot{q}$  w/o insulation?

~~$$\frac{\dot{q}}{L} = k_{\text{ins}} \frac{A_s}{L} (T_i - T_\infty)$$~~

$$\dot{q} = h A_s (T_i - T_\infty)$$

$$\frac{\dot{q}}{L} = 3 \frac{\text{W}}{\text{m}^2\cdot\text{K}} \times 2\pi (0.025 \text{ m}) (200^\circ\text{C} - 20^\circ\text{C})$$

$$= 84.8 \frac{\text{W}}{\text{m}}$$

- what is  $\dot{q}$  w/  $r_{\text{crit}}$  of insulation

$$r_{\text{crit}} = \frac{k_{\text{ins}}}{h} = \frac{0.17 \frac{\text{W}}{\text{m}\cdot\text{K}}}{3 \frac{\text{W}}{\text{m}^2\cdot\text{K}}} = 0.0567 \text{ m}$$

~~$$\frac{\dot{q}}{L} = k_{\text{ins}} A_s \frac{(T_i - T_2)}{r_{\text{crit}} - r_i} = 0.17 \frac{\text{W}}{\text{m}\cdot\text{K}} \times 2\pi ( ) \frac{(200^\circ\text{C} - T_2)}{(0.0567 - 0.025) \text{ m}}$$~~

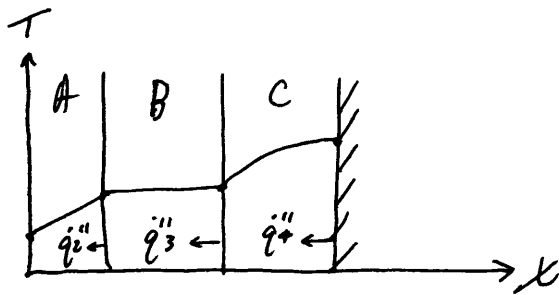
$$T_i \text{ --- } \frac{1}{2\pi k_{\text{ins}} L \ln(r_{\text{crit}}/r_i)} \text{ --- } \frac{1}{h 2\pi r_{\text{crit}} L} \text{ --- } T_\infty$$

$$\frac{\dot{q}}{L} = \frac{T_i - T_\infty}{\sum R_{\text{th}}} = 106 \frac{\text{W}}{\text{m}}$$

# Internal Heat Generation

$$\dot{q} = KA \frac{\Delta T}{\Delta x} \rightarrow \Delta T(\Delta x) = \frac{\dot{q}}{KA} \Delta x$$

S.S. heat generation in a plane wall of three different materials, each w/ constant  $K$ .



a) comment on the relative magnitudes of  $\dot{q}_1''$ ,  $\dot{q}_2''$ , &  $\dot{q}_3''$

b) comment on relative magnitudes of  $K_A, K_B$ , &  $K_C$

c) sketch  $\dot{q}''(x)$

$$T(x) = \frac{\dot{q}}{K} \frac{1}{A} x \quad \dot{q}'' = K \frac{\Delta T}{\Delta x} \rightarrow T(x) = \frac{\dot{q}''}{K} x$$

a)  $\dot{q}_3'' = 0$

Inside C:  $\dot{q}''' \neq 0 \rightarrow$  internal heat generation

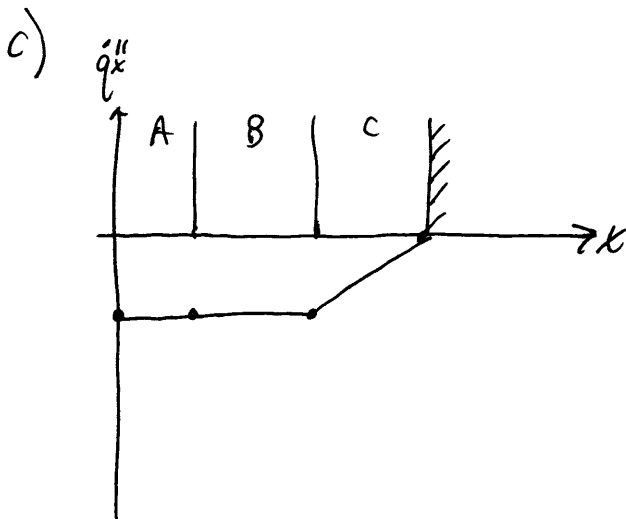
The curve in C indicates internal heat generation

$$\dot{q}_2'' = \dot{q}_3''$$

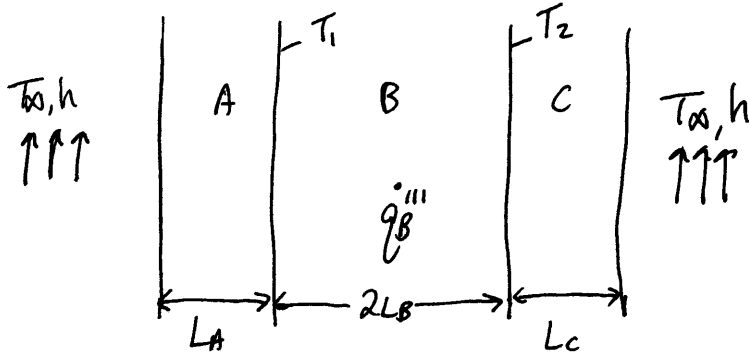
b)  $K_A < K_B \quad K_B > K_C$

$$\dot{q}'' = \dot{q}_2'' = \dot{q}_3''$$

$$K_A \frac{dT}{dx_A} = K_B \frac{dT}{dx_B}$$



3.73)



$T_\infty = 25^\circ\text{C}$  internal heat generat.

$h = 1000 \frac{\text{W}}{\text{m}^2\cdot\text{K}}$

$\dot{q}_B'''$

$T_1 = 261^\circ\text{C}, T_2 = 211^\circ\text{C}$

$K_A = 25 \frac{\text{W}}{\text{m}\cdot\text{K}}$

$L_A = 30\text{mm}$

$K_C = 50 \frac{\text{W}}{\text{m}\cdot\text{K}}$

$L_B = 30\text{mm}$

$L_C = 20\text{mm}$

a) assuming negligible contact resistance at the interfaces, determine  $\dot{q}_B''' \frac{1}{K_B}$

$$\dot{q}''' = K \frac{1}{L} \frac{dT}{dx} = \frac{\dot{q}''}{L} \rightarrow \dot{q}'' = \dot{q}''' \times L \rightarrow \boxed{\dot{q}_B''' L_B = \dot{q}_1'' + \dot{q}_2''}$$

$$\dot{q}_1'' = K_A \frac{T_1 - T_\infty}{L_A} = 25 \frac{(261 - 25)}{0.03} = 197 \frac{\text{KW}}{\text{m}^2}$$



$$\dot{q}_1'' = \frac{T_1 - T_\infty}{\sum R_{th}} = \frac{534\text{K} - 298\text{K}}{\frac{1}{1000 \frac{\text{W}}{\text{m}^2\cdot\text{K}}} + \frac{0.03\text{m}}{25 \frac{\text{W}}{\text{m}\cdot\text{K}}}} = 107 \frac{\text{KW}}{\text{m}^2}$$

$$\dot{q}_2'' = \frac{484\text{K} - 298\text{K}}{\frac{0.02\text{m}}{50 \frac{\text{W}}{\text{m}\cdot\text{K}}} + \frac{1}{1000 \frac{\text{W}}{\text{m}^2\cdot\text{K}}}} = 133 \frac{\text{KW}}{\text{m}^2}$$

$$\dot{q}_B''' = \frac{\dot{q}_1'' + \dot{q}_2''}{2L_B} = \frac{107,000 + 133,000}{2(0.03)} = \boxed{4 \frac{\text{MW}}{\text{m}^3}}$$

$$2L_B \dot{q}_B''' = K_B \frac{dT}{dx} = K_B \frac{dT}{dx}$$

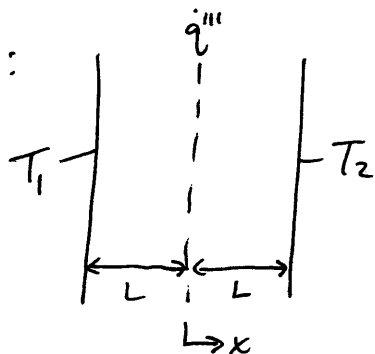
$$T(x) = \frac{-\dot{q}''' x^2}{2K_B} + C_1 x + C_2$$

$$\left[ \frac{\partial T(x)}{\partial x} \right] = \left[ \frac{-\dot{q}''' x}{K_B} + C_1 \right] K_B = \dot{q}''' x - C_1 K_B$$

# Recap: Int. Heat Generation

S.S.,  $\dot{q}''' = \text{const.}$ ,  $k = \text{const.}$

Planar Systems:



$$\frac{d^2T}{dx^2} + \frac{\dot{q}'''}{k} = 0$$

$$T = \frac{-\dot{q}''' x^2}{2k} + C_1 x + C_2$$

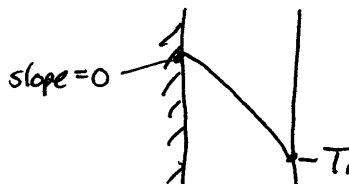
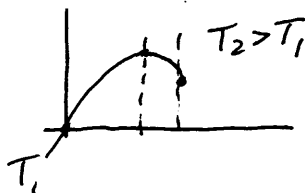
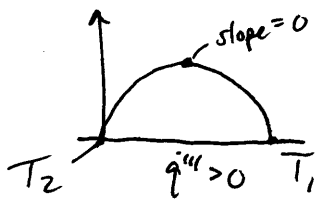
$C_1, C_2$  obtained from B.C.

$$T(x) = \frac{\dot{q}''' L^2}{2k} \left(1 - \frac{x^2}{L^2}\right) + \frac{T_2 - T_1}{2} \frac{x}{L} + \frac{T_1 + T_2}{2}$$

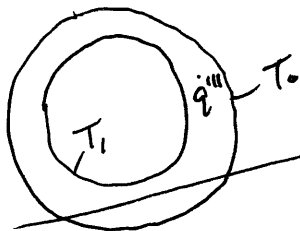
$$T_1 = T_2 = T$$

$$T(x) = \frac{\dot{q}''' L^2}{2k} \left(1 - \frac{x^2}{L^2}\right) + T$$

T Profiles



~~Radial Systems~~



~~$$T = T_0 + \frac{\dot{q}'''}{4k} (r_0^2 - r_i^2) + C \frac{r}{r_0}, \quad C = \frac{T_1 - T_0 + \dot{q}''' (r_0^2 - r_1^2) / 4k}{\ln(r_0/r_1)}$$~~

## Cylinders w/ Heat Generation

- Uniformly distributed heat sources
- constant conductivity

$$\frac{d^2T}{dr^2} + \frac{1}{r} \frac{dT}{dr} + \frac{\dot{q}'''}{k} = 0 \quad - \text{gov. eq.}$$

$$\text{B.C.: } T = T_w @ r = R, \quad \frac{dT}{dr} = 0 @ r = 0$$

Alt. B.C.: Heat flux at surface = energy generated within the solid

$$\dot{q}''' \pi R^2 L = -k 2\pi R L \left. \frac{dT}{dr} \right|_{r=R}$$

$$r \frac{d^2T}{dr^2} + \frac{dT}{dr} = -\frac{\dot{q}''' r}{k}$$

$$\text{-or-} \quad \frac{d}{dr} \left( r \frac{dT}{dr} \right) = \frac{-\dot{q}''' r}{k} \quad \xrightarrow{\text{integrate}} \quad r \frac{dT}{dr} = \frac{-\dot{q}''' r^2}{2k} + A$$

$$\text{B.C.: } 0 = \frac{dT}{dr} @ r = 0 \quad \rightarrow A = 0$$

$$\xrightarrow{\text{integrate}} \quad T = \frac{-\dot{q}''' r^2}{4k} + B$$

$$\text{B.C.: } \text{At } r = R, T = T_w$$

$$\boxed{T - T_w = \frac{\dot{q}'''}{4k} (R^2 - r^2)}$$

Non-dimensional:

$$\boxed{\frac{T - T_w}{T_0 - T_w} = 1 - \left(\frac{r}{R}\right)^2} \quad T_0 = T(r=0) = \frac{\dot{q}''' R^2}{4k} + T_w$$

## Hollow Cylinder

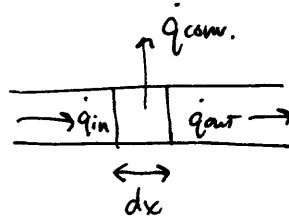
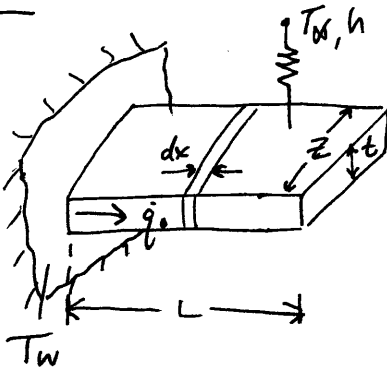
$$\text{B.C.: } T = T_i @ r = r_i, \quad T = T_o @ r = r_o$$

$$\boxed{T - T_o = \frac{\dot{q}'''}{4k} (r_o^2 - r^2) + A \ln\left(\frac{r}{r_o}\right)}$$

$$\boxed{A = \frac{T_i - T_o + \dot{q}''' (r_i^2 - r_o^2) / 4k}{\ln(r_i / r_o)}}$$

9/28/09

Fins



Energy Balance:  $q_{in} = q_{out} + q_{conv}$

$$\left\{ \begin{aligned} q_{in}(x) &= -kA \frac{dT}{dx} = -k(zt) \frac{dT}{dx} \\ q_{out}(x) &= q_{in} + \frac{d}{dx}(q(x)) dx && \text{- Taylor Expansion} \\ q_{conv} &= hA_s \Delta T = h(\underbrace{2z+2t}_{\text{perim.}}) dx (T(x) - T_{\infty}) \end{aligned} \right.$$

E. Balance:  $q_{in}(x) = q_{in} + \frac{d}{dx}(q(x)) dx + hP dx (T(x) - T_{\infty})$

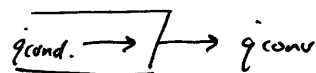
solve to get:  $\boxed{\frac{d^2T}{dx^2} - \frac{hP}{kA} (T - T_{\infty}) = 0}$

Some B.C.

1) Fin is infinitely long:  $T$  at tip =  $T_{\infty}$

2) Fin tip is insulated:  $\frac{dT}{dx} = 0$  @  $x=L$

3) Fin loses heat by convection at tip:



$$\begin{aligned} q_{cond.} &= q_{conv.} \\ -kA \frac{dT}{dx} \Big|_{x=L} &= hA (T_L - T_{\infty}) \end{aligned}$$

General Solution to Gov. Eq :

$$\text{Define } \theta = T - T_{\infty} \rightarrow \frac{d\theta}{dx} = \frac{dT}{dx}, \quad \frac{d^2\theta}{dx^2} = \frac{d^2T}{dx^2}$$

$$\rightarrow \frac{d^2\theta}{dx^2} - \frac{hP}{KA} \theta = 0$$

$$\text{Define } m^2 = \frac{hP}{KA} \rightarrow \boxed{\frac{d^2\theta}{dx^2} - m^2\theta = 0}$$

$$\text{Try } \theta = \sin(mx), \quad \frac{d\theta}{dx} = m\cos(mx), \quad \frac{d^2\theta}{dx^2} = -m^2\sin(mx)$$

$$\text{Try } \theta = e^{mx}, \quad \frac{d\theta}{dx} = me^{mx}, \quad \frac{d^2\theta}{dx^2} = m^2e^{mx}$$

$$\text{gen. sol. : } \boxed{\theta = C_1 e^{mx} + C_2 e^{-mx}}$$

Case 1) Assume fin is infinitely long :  $T \rightarrow T_{\infty}$  @  $x \rightarrow \infty$

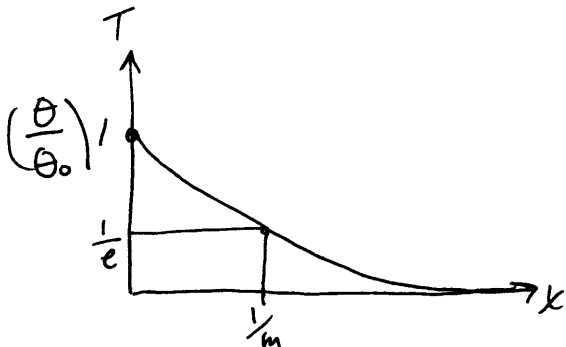
$$T = T_{\infty} \rightarrow \theta = T - T_{\infty} = 0$$

$$0 = C_1 e^{m\infty} + C_2 e^{-m\infty} \quad C_1 = 0$$

$$\text{@ } x=0, T = T_w \rightarrow \theta_0 = T_w - T_{\infty}$$

$$\theta_0 = C_2 e^{-m \cdot 0} \rightarrow C_2 = \theta_0$$

$$\rightarrow \theta = \theta_0 e^{-mx} \quad \text{or} \quad \boxed{\frac{\theta}{\theta_0} = e^{-mx}} \quad \text{non-dimensional}$$



Case 2) Tip is insulated :  $\frac{dT}{dx}\bigg|_{x=L} = 0$  ,  $T = T_w @ x=0$   
 - solve for  $C_1, C_2$

$$\boxed{\frac{\Theta}{\Theta_0} = \frac{e^{-mx}}{1+e^{-2mL}} + \frac{e^{mx}}{1+e^{2mL}} = \frac{\cosh(m(L-x))}{\cosh(mL)}}$$

Case 3) convection at tip :  $KA \frac{dT}{dx}\bigg|_{x=L} = hA(T-T_w)$

$$\frac{T-T_w}{T_w-T_w} = \frac{\Theta}{\Theta_0} = \frac{\cosh m(L-x) + \left(\frac{h}{mk}\right) \sinh m(L-x)}{\cosh mL + \frac{h}{mk} \sinh mL}$$

$$\dot{q}_0 = -KA \frac{dT}{dx}\bigg|_{x=0} \quad \dot{q}_0 \rightarrow \boxed{\phantom{000}}$$

To find heat flux : compute  $\dot{q}_0 = -KA \frac{dT}{dx}\bigg|_{x=0}$

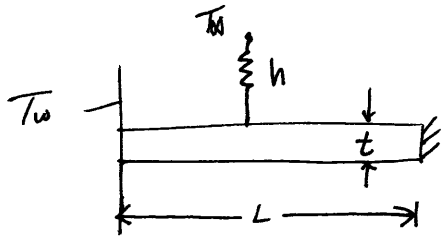
B.C. 1 :  $\dot{q}_0 = \Theta_0 \sqrt{hPKA}$

B.C. 2 :  $\dot{q}_0 = \sqrt{hPKA} \Theta_0 \tanh(mL)$

B.C. 3 :  $\dot{q}_0 = \sqrt{hPKA} (T_w - T_w) \frac{\sinh(mL) + \frac{h}{mk} \cosh(mL)}{\cosh(mL) + \frac{h}{mk} \sinh(mL)}$



ex) Aluminum Fin:  $k = 200 \frac{W}{m \cdot K}$ , 3 mm thick, 7.65 mm long, rect. c.s.  
 infinitely ~~long~~ deep,  $T_s = 58^\circ C$ ,  $h = 10 \frac{W}{m^2 \cdot K}$ ,  $T_w = 300^\circ C$



- case 2

$$z \rightarrow \infty, 2t \rightarrow 0$$

$$M = \sqrt{\frac{hP}{KA}} = \sqrt{\frac{h(2z+2t)}{k(2t)}} = \sqrt{\frac{h(2z)}{k(2t)}} = \sqrt{\frac{2h}{kt}} = 5.77 \frac{1}{m}$$

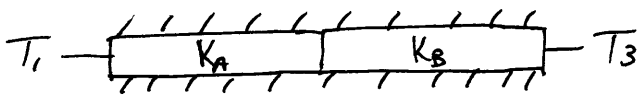
$$\dot{q}_0 = \tanh(mL) \sqrt{hPKA} \theta_0$$

$$hPKA = h(2z+2t)k(2t) = 2hkz^2t$$

$$\frac{\dot{q}_0}{2} = \dot{q}'_0 = \tanh(mL) \sqrt{2hkt} \theta_0 \rightarrow \underline{\underline{\dot{q}'_0 = 360 \frac{W}{m}}}$$

Sect. 3.1.4 - contact resistance

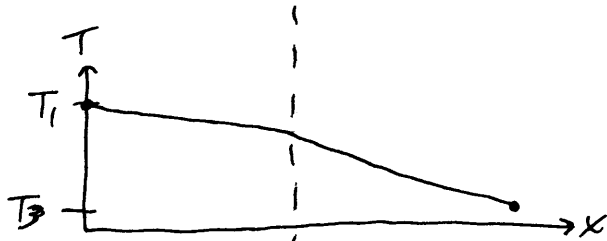
two rods in contact with each other



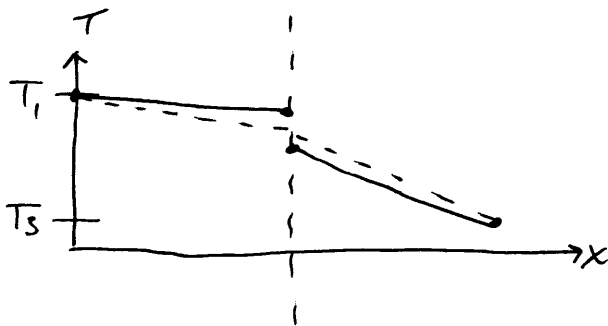
$$K_A > K_B$$

$$T_1 > T_3$$

$$\dot{q}''' = 0, \text{ S.S.}$$



- Ideal temp. profile



- Actual temp. profile



- Jump in temp. due to contact resistance.

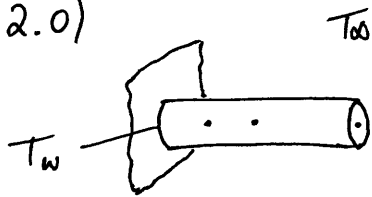
$$\dot{q}'' = K_A \frac{T_1 - T_{2A}}{\Delta x_A} = K_B \frac{T_{2B} - T_3}{\Delta x_B} = \frac{T_{2A} - T_{2B}}{1/h_c}$$

$$\frac{1}{h_c} = \text{contact resistance}$$

$$\frac{1}{h_c} = R_{t,c}'' \text{ (book)}$$

$$\left[ \frac{\text{m}^2 \cdot \text{K}}{\text{W}} \right]$$

3.1.2.0)



$$L = 100 \text{ mm}$$

$$d = 5 \text{ mm}$$

$$k = 133 \frac{\text{W}}{\text{m}\cdot\text{K}}$$

$$T_w = 200^\circ\text{C}$$

$$T_\infty = 20^\circ\text{C}$$

$$h = 30 \frac{\text{W}}{\text{m}^2\cdot\text{K}}$$

$$T_{25\text{mm}} = ?$$

$$T_{50\text{mm}} = ?$$

$$T_{100\text{mm}} = ?$$

$$m = \sqrt{\frac{hP}{kA_c}} = \sqrt{\frac{h\pi d}{k\frac{\pi}{4}d^2}} = 13.43 \text{ m}^{-1}$$

$$\frac{h}{mk} = 0.0168$$

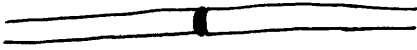
$$\frac{\theta}{\theta_0} = \frac{\cosh m(L-x) + \left(\frac{h}{mk}\right) \sinh m(L-x)}{\cosh mL + \frac{h}{mk} \sinh mL}$$

$$\theta = T - T_\infty$$

$$\theta_0 = T_w - T_\infty$$

$$T(x) = \frac{\cosh m(L-x) + \left(\frac{h}{mk}\right) \sinh m(L-x)}{\cosh mL + \frac{h}{mk} \sinh mL} (T_w - T_\infty) + T_\infty$$

3.124) Two long copper rods  
soldered together



$T_{\infty}$

$d = 10 \text{ mm}$   
Melting Pt<sub>solder</sub> =  $650^{\circ}\text{C}$

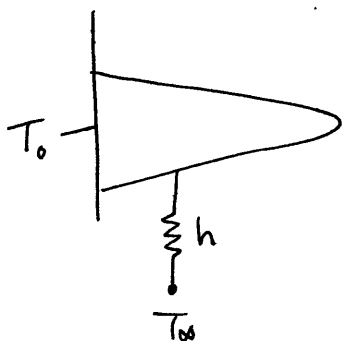
$T_{\infty} = 25^{\circ}\text{C}$   
 $h = 10 \frac{\text{W}}{\text{m}^2\text{-K}}$   
 $K = 379 \frac{\text{W}}{\text{m-K}}$   
 $P_{\text{min}} = ?$

$$\text{Heat flux: } q_f = \sqrt{hPKA_c} (T - T_{\infty}) \times 2 \text{ rods}$$

$$= 2\sqrt{h\pi d K \frac{\pi}{4} d^2} (T - T_{\infty}) = 120.9 \text{ W}$$

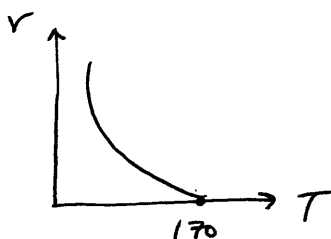
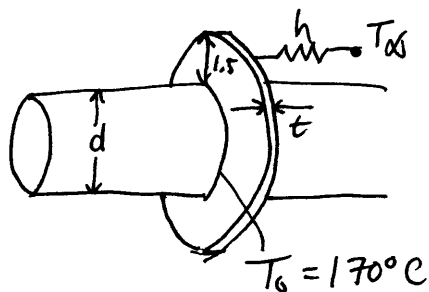
### Fin Efficiency :

$$\eta_{fin} = \frac{\text{actual heat transfer}}{\text{maximum possible H.T. by fin}} = \frac{\dot{q}_f}{h A_f (T_o - T_{\infty})}$$



$\eta$  values - Fig 3.18, 3.19, Table 3.5

- ex) Aluminum fin 1.5 cm wide, 1.0 mm thick, on a 2.5 cm diameter tube. Tube surface at 170°C,  $T_{\infty} = 25^\circ\text{C}$ ,  $h = 130 \frac{\text{W}}{\text{m}^2\cdot\text{K}}$ ,  $K_{Al} = 200 \frac{\text{W}}{\text{m}\cdot\text{K}}$ . H.T. per fin?



$$\dot{q}_{max} = h A_f (T_o - T_{\infty}) = (130 \frac{\text{W}}{\text{m}^2\cdot\text{K}}) [2\pi (r_o^2 - r_i^2) + 2\pi r_o t] (170^\circ\text{C} - 25^\circ\text{C})$$

To find  $\eta_f$ , <sup>fig.</sup> ~~table~~ 3.19

$$r_{2c} = r_2 + \frac{t}{2} = r_o + \frac{t}{2}$$

$$L_c = L + \frac{t}{2} = (r_o - r_i) + \frac{t}{2}$$

$$L_c^{3/2} \left( \frac{h}{K A_f} \right)^{1/2} = A_p = L_c t$$

$$L_c = L + \frac{t}{2} = 1.5 \text{ cm} + \frac{1 \text{ mm}}{2} = 0.0155 \text{ m}$$

$$L_c^{3/2} \left( \frac{h}{K A_f} \right)^{1/2} = 0.395$$

$$r_{2c} = r_i + L_c = 1.25 \text{ cm} + 1.55 \text{ cm} = 0.0280 \text{ m}$$

$$\frac{r_{2c}}{r_i} = 2.24$$

$$A_p = t (r_{2c} - r_i) = (1 \times 10^{-3}) (0.028 - 0.0125) = 1.55 \times 10^{-5} \text{ m}^2$$

From 3.19,  $\eta_f = 0.82$

$$\eta_{\text{effectiveness}} = \frac{\text{heat transfer w/ fin}}{\text{heat transfer w/o fin}} = \frac{\eta_f A_f h (T_o - T_{\infty})}{h A_b (T_o - T_{\infty})}$$

## Chapter 4

### 2-D Conduction

10/5/09

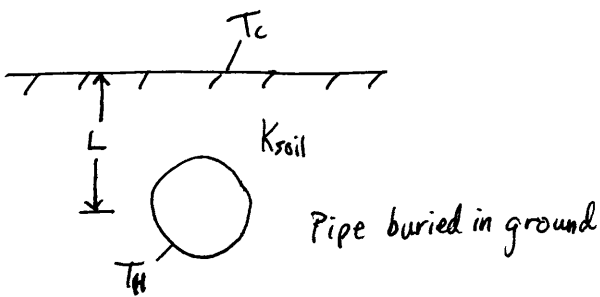
Conduction shape factor:

Applicable when the temperature of the body & surroundings are specified

$$\dot{q} = k S \Delta T_{\text{overall}}$$

thermal conductivity of medium

S - conduction shape factor  
Table A-1



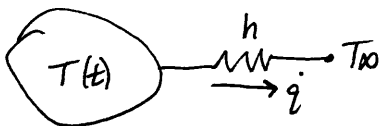
### Unsteady Heat Conduction

Temperatures can change w/ time

- Lumped capacity (Uniform temperature through-out, but changing w/ time)
- semi-infinite solid
- multi-dimensional conduction

### Lumped Capacity

- Temperature in body is spatially uniform, but changing w/ time.



Heat loss by convection results in temp drop in body.

$$\dot{q} = h A_s (T(t) - T_{\infty}) = -m C_p \frac{dT}{dt} = -\rho V C_p \frac{dT}{dt}$$

$$B.C.: T = T_i \text{ @ } t = 0$$

Separate Variables:

$$\frac{dT}{T - T_\infty} = -\frac{hA_s}{mC_p} dt$$

$$\ln(T - T_\infty) = -\frac{hA_s}{\rho V C_p} t + C_1$$

$$T - T_\infty = \exp\left(-\frac{hA_s}{\rho V C_p} t\right) C_2$$

$$I.C.: T = T_i \text{ @ } t = 0 \rightarrow C_2 = T_i - T_\infty$$

$$\boxed{\frac{T - T_\infty}{T_i - T_\infty} = \exp\left[-\frac{hA_s}{\rho V C_p} t\right]}$$

$$\frac{T - T_\infty}{T_i - T_\infty}$$



- If conduction in body is large,  $R_{th}$  in body is small and can be neglected, so Temp is uniform

$$\dot{q}'' \sim k \frac{T_i - T_\infty}{L} \sim h (T_o - T_\infty)$$

$$\rightarrow T_i - T_o \sim \frac{hL}{k} (T_o - T_\infty + T_i - T_i)$$

$$\sim B_1 (T_o - T_i) + B_2 (T_i - T_\infty)$$

$$\rightarrow \frac{T_i - T_o}{T_i - T_\infty} = \frac{B_1}{1 + B_1}$$

- For lumped capacity,  $B_1 \ll 1$

- For most geometries, use  $L = \frac{V}{A_s}$ . If  $B < 0.1$ , solutions will be within 5%

### Example

Steel ball bearing:  $C_p = 0.46 \frac{kJ}{kg \cdot K}$ ,  $k = 35 \frac{W}{m \cdot K}$ ,  $D = 5 \text{ mm}$ ,  $T_i = 450^\circ C$ ,  $h = 10 \frac{W}{m^2 \cdot K}$ ,  $T_\infty = 100^\circ C$

How long until  $T = 150^\circ C$ ?

$$B_1 = \frac{h(V/A_s)}{k} = \frac{h\left(\frac{4}{3}\pi r^3 / 4\pi r^2\right)}{k} = \frac{h(r/3)}{k} \rightarrow B_1 = 0.0024 < 0.1 \quad \therefore \text{lumped capacity}$$

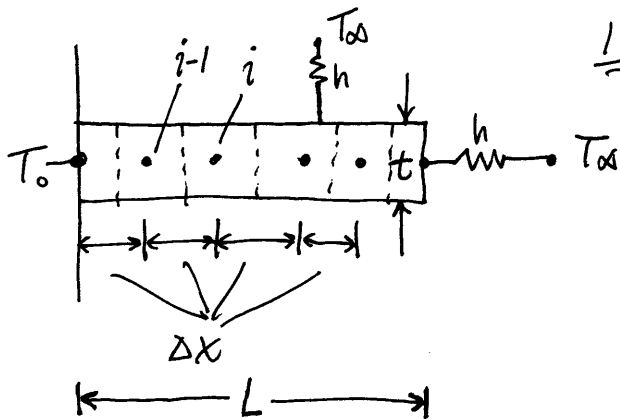
$$\frac{T - T_\infty}{T_i - T_\infty} = \exp\left(\frac{-hA_s}{\rho C_p V} t\right), \quad \frac{hA_s}{\rho V C_p} = 3.34 \times 10^{-4} \text{ s}^{-1}$$

$$\frac{T - T_\infty}{T_i - T_\infty} = 0.143 = \exp(-3.34 \times 10^{-4} t)$$

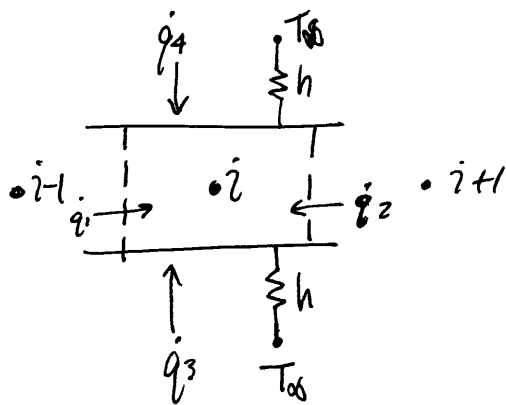
$$\boxed{t = 5826 \text{ s}}$$

# Numerical Method for Heat Conduction

1-D Fin



- 1) Assign nodes on fin
- 2) Draw control forces around each node that are equally spaced between nodes
- 3) Perform energy balance on each node



Energy balance on node  $i$ :  $q_1 + q_2 + q_3 + q_4 = 0$

$$kt \frac{T_{i-1} - T_i}{\Delta x} + kt \frac{T_{i+1} - T_i}{\Delta x} + 2h\Delta x(T_{\infty} - T_i) = 0$$

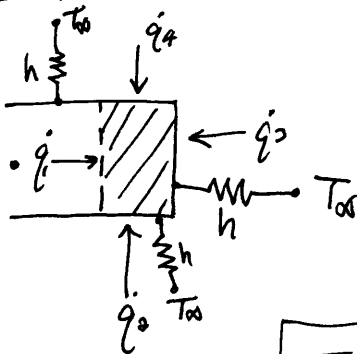
$\underbrace{\hspace{10em}}_{q_1} \quad \underbrace{\hspace{10em}}_{q_2} \quad \underbrace{\hspace{10em}}_{q_3 + q_4}$

Solve for  $T_i$ :  $T_i \left[ 2 \frac{kt}{\Delta x} + 2h\Delta x \right] = \frac{kt}{\Delta x} (T_{i-1} + T_{i+1}) + 2h\Delta x T_{\infty}$

$$\boxed{T_i = \frac{\frac{kt}{\Delta x} (T_{i-1} + T_{i+1}) + 2h\Delta x T_{\infty}}{2 \frac{kt}{\Delta x} + 2h\Delta x}}$$

} node eq. for node  $i$

At tip:



$$\sum q_i = 0$$

$$kt \frac{T_{n-1} - T_n}{\Delta x} + h \frac{\Delta x}{2} (T_{\infty} - T_n) + h \frac{\Delta x}{2} (T_{\infty} - T_n) + ht(T_{\infty} - T_n) = 0$$

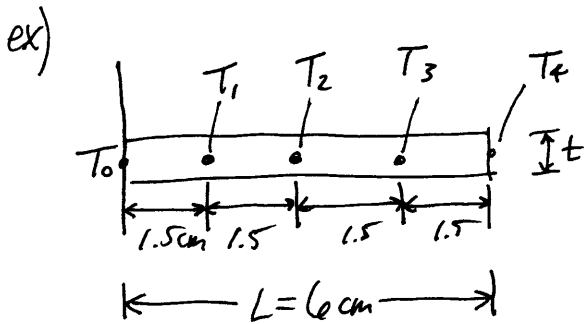
Solve for  $T_n$ :

$$\boxed{T_n = \frac{\frac{kt}{\Delta x} T_{n-1} + ht T_{\infty} + h\Delta x T_{\infty}}{\frac{kt}{\Delta x} + ht + \Delta x h}}$$



Can solve these eq.s to get  $T_i$ 's using Gauss-Seidel

- 1) Guess values for  $T_i$
- 2) Use node eqs to update  $T_i$ 's.
- 3) Iterate until convergence



$$T_0 = 100^\circ\text{C}$$

$$\Delta x = 1.5\text{ cm}$$

$$t = 0.002\text{ m}$$

$$k = 170 \frac{\text{W}}{\text{m}\cdot\text{K}}$$

$$h = 200 \frac{\text{W}}{\text{m}^2\cdot\text{K}}$$

$$T_\infty = 0$$

Node eqs for interior nodes  $T_1, T_2, T_3$ :

$$T_i = \frac{\frac{kt}{\Delta x} (T_{i-1} + T_{i+1})}{\frac{2kt}{\Delta x} + 2h\Delta x} = \underline{\underline{0.442(T_{i-1} + T_{i+1})}}$$

Node eqn at tip:

$$T_n = \frac{\frac{kt}{\Delta x} T_{n-1}}{\frac{kt}{\Delta x} + h t + \Delta x h} \rightarrow \underline{\underline{T_n = 0.869 T_{n-1}}}$$

Gauss-Seidel:  $T_\infty = 0$

- 1) Assume  $T_1 = T_2 = T_3 = T_4 = 0$
  - 2)  $T_1 = 0.442(100 + 0) = 44.2$   $T_2 = 0$
  - 3)  $T_2 = 0.442(T_1 + T_3) = 0.442(44.2 + 0) = 19.5$   $T_2 = 19.5$
  - 4)  $T_3 = 0.442(T_2 + T_4) = 8.67$   $T_4 = 0$
  - 5)  $T_4 = 0.869 T_3 = 7.58$
- } 1<sup>st</sup> iteration

Excel:

$T_0$	$T_1$	$T_2$	$T_3$	$T_4$
100	0	0	0	0