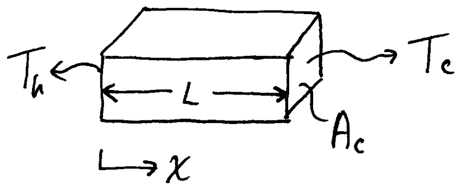
ENME332 - Heat Transfer

8/31/09

3 modes of heat transfer: conduction, convection, radiation

conduction - molecular vibrations / electronic transfer



$\underset{[W]}{\dot{q}} \propto \frac{\Delta T}{L} A_c$ $\qquad \Delta T = T_h - T_c$

so we expect that $\boxed{\dot{q} = k A_c \frac{\Delta T}{L}}$ Fourier's Law

$\dot{q} = k A_c \frac{dT}{dx}$ as $L \to 0$

units: $[\dot{q}] = W = J/s$

$[A_c] = m^2$

$\left[\frac{dT}{dx}\right] = \frac{°C}{m} = \frac{K}{m}$

$[k] = \left[\frac{q}{A_c \frac{dT}{dx}}\right] = \frac{W}{m\text{-}K} = \frac{W}{m\text{-}°}$

nomenclature:

$q\,[W]$ $\qquad q'\,[W/m]$

$\dot{q}\,[W/m^3]$ $\qquad q''\,[W/m^2]$ $\qquad$ (book)

overdot = "per unit time"

prime = "per unit length"

professor's		book's
$\dot{q}\,[W] \equiv [J/s]$	$\equiv$	q
$\dot{q}'''\,[W/m^3]$	$\equiv$	$\dot{q}$
$\dot{q}'\,[W/m]$	$\equiv$	q'
$\dot{q}''\,[W/m^2]$	$\equiv$	q''

property values in book:

thermal conductivity of air:

air properties

T (K)	$K \times 10^3$ $\frac{W}{m\text{-}K}$
100	~
200	~
300	26.3

$K_{air} = 26.3 \times 10^{-3} \frac{W}{m}$

$K_{air} \neq 26.3 \times 10^{3} \frac{W}{m\text{-}K}$

gases:

- K_{gases} is independent of pressure

$K_{air} = 0.026 \frac{W}{m\text{-}K}$ $\qquad K_{helium} = 0.175 \frac{W}{m\text{-}K}$

$K_{hydrogen} = 0.175 \frac{W}{m\text{-}K}$ $\qquad K_{CO_2} = 0.015 \frac{W}{m\text{-}K}$

liquids:

- $K_{liquids}$ depends on temperature, but can increase/decrease depending on fluid & temperature

$K_{water} = 0.556 \frac{W}{m\text{-}K}$

oil $= 0.147 \frac{W}{m\text{-}K}$

mercury $= 8.21 \frac{W}{m\text{-}K}$

solids:

- lattice vibration electronic transport

K_{glass} : $1.5 \frac{W}{m\text{-}K}$

$K_{stainless\ steel}$: $15 \frac{W}{m\text{-}K}$

$K_{silicon}$: $120 \frac{W}{m\text{-}K}$

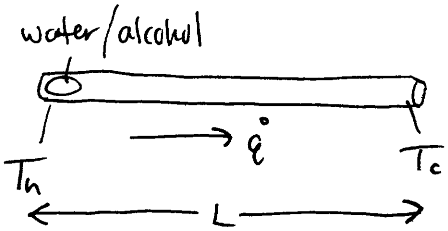
$K_{sapphire}$: $70 \frac{W}{m\text{-}K}$

$K_{aluminum}$: $200 \frac{W}{m\text{-}K}$

K_{copper} : $400 \frac{W}{m\text{-}K}$

$K_{diamond}$: $2000 \frac{W}{m\text{-}K}$

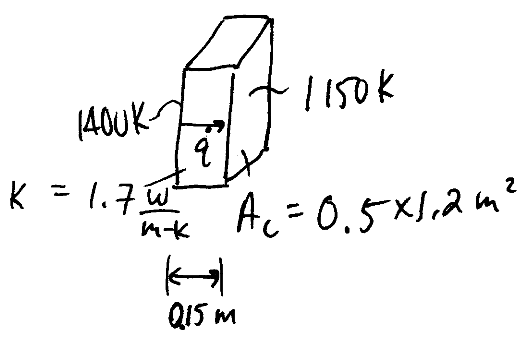
heat pipes: $K_{effective} \sim 20{,}000 \frac{W}{m\text{-}K}$



ex) walls of a furnace made of brick ($k = 1.7 \frac{W}{m\text{-}K}$) wall is 0.15 m thick

$T_{inner} = 1400$ K, $T_{outer} = 1150$ K

what is heat loss over a $0.5 \times 1.2\ m^2$ area?

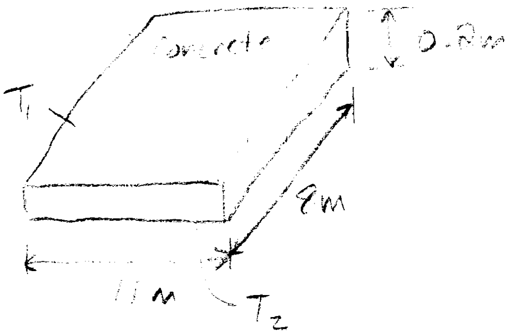


$$\dot{q} = kA\frac{dT}{dx}$$

$$= \left(1.7 \frac{W}{m\text{-}K}\right)(0.5m \times 1.2m)\left(\frac{-1400K + 1150K}{0.15m}\right)$$

$$= -1700\ W$$

1.3)



$k = [illegible] \frac{W}{m \cdot K}$

$T_1 = 17°C$

$T_2 = 10°C$

$\eta_f = 0.9$ efficiency

cost = \$0.01/MJ

$$\dot{q} = -k A_c \frac{\Delta T}{L}$$

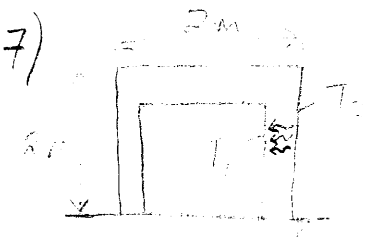
$$= -[illegible] \frac{W}{m \cdot K} (11m \times 8m) \frac{(10-17)°C}{0.2\,m} = 4312\ W$$

$$\eta_f = \frac{\dot{q}_o}{\dot{q}_i} \rightarrow \dot{q}_i = \frac{\dot{q}_o}{\eta_f} \longrightarrow \dot{q}_i = \frac{4312\ W}{0.9} = 4,791\ W$$

1 day = 86,400 s $\longrightarrow$ (4,791 J/s)(86400 s) = 414 MJ in 1 day

$$\$0.01/MJ\,(414\ MJ) = \boxed{\$4.14 \text{ per day}}$$

1.7)



$T_1 = -10°C$

$T_2 = 35°C$

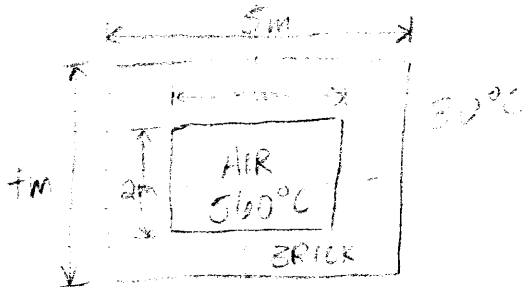
$k = 0.03 \frac{W}{m \cdot K}$

$\dot{q} = 500\ W$

$$\dot{q} = -k A_c \frac{\Delta T}{L}$$

$$500\ W = \left(-0.03 \frac{W}{m \cdot K}\right)(5 \times 2m [illegible]) \frac{(-10-35)°C}{t}$$

$$\boxed{t = 0.054\ m}$$



$$\dot{Q} = -h A_c \Delta T$$

heat transfer coefficient
for convection

$h_{inside} = 10$

$h_{outside} = 5$

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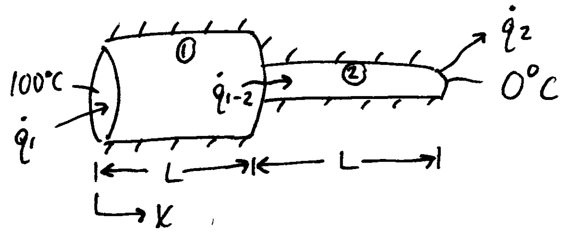
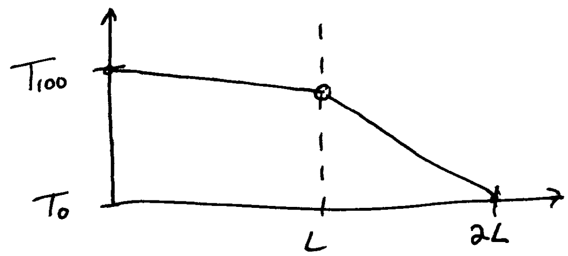
example for <u>Heat Conduction</u>:

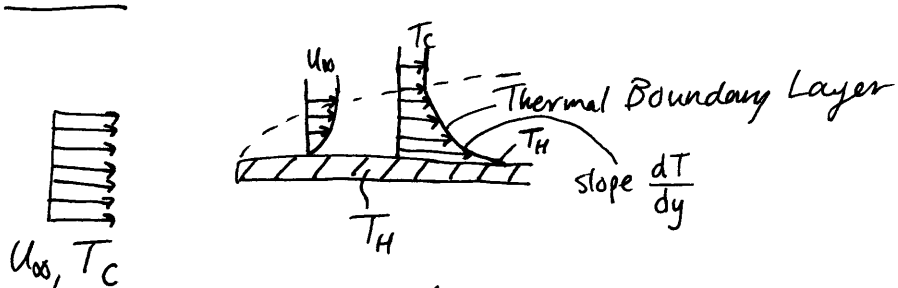
$$\dot{q} = -kA\frac{dT}{dx}$$





<u>Steady state</u>: all heat flowing must flow out

<u>Convection</u> - Heat transfer by bulk fluid motion

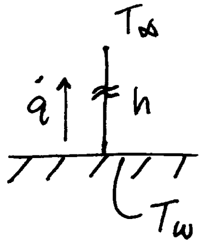


$$\dot{q} = -kA\frac{dT}{dy}$$

$$* \quad \dot{q} = hA_s(T_w - T_\infty)$$

surface area available for convection

heat transfer coefficient $[h] = \left[\frac{-\dot{q}}{A_s(T_w - T_\infty)}\right] = \frac{W}{m^2\text{-}K} \equiv \frac{W}{m^2\text{-}°C}$



$$\dot{q} = hA(T_w - T_\infty)$$

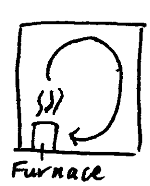
or $\dot{q}'' = \frac{\dot{q}}{A} = h(T - T_\infty)$

$\left(\frac{W}{m^2}\right)$ Heat Flux

<u>Types of Convection</u>:

Forced Convection - Fluid forced over a surface (wind tunnel) (Boiling & condensation

Free Convection - (Natural convection) ⇒



Free Convection in air : $h \cong 6-10 \ \frac{W}{m^2\text{-}K}$

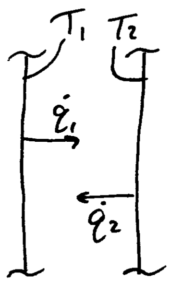
" " " water : $h \cong 1000 \ \frac{W}{m^2\text{-}K}$

Forced Convection in air : $h \cong 10-100 \ \frac{W}{m^2\text{-}K}$

" " " Water : $h \sim 3000 \ \frac{W}{m^2\text{-}K}$

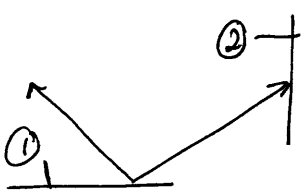
Pool Boiling in water : $2500 - 35{,}000 \ \frac{W}{m^2\text{-}K}$

Suppose we had two infinite plates facing each other :



The net radiation exchange between 1 & 2 is given by :

What if surfaces don't see each other completely?



use a view factor to account for this

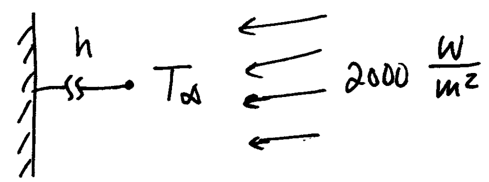
$$\dot{q}''_{1-2} = \sigma \underbrace{F_{ij}}_{\text{view factor}} (T_1^4 - T_2^4)$$

Non-black body behavior is accounted for by an "emissivity"

$$\dot{q}''_{1-2} = \varepsilon \sigma F_{ij} (T_1^4 - T_2^4)$$

$0 < \varepsilon < 1 \qquad \varepsilon = 1 \Rightarrow \text{black body}$

ex) coating on a plate is cured by exposure to IR radiation that provides uniform illumination @ $2000 \frac{W}{m^2}$. The surface is also exposed to an air flow @ $20°C$. The surroundings are at $30°C$. If the surface absorbs 80% of incoming radiation and has $\varepsilon = 0.50$, what is the temperature of the surface if $h = 15 \frac{W}{m^2 \cdot K}$?



* Assume S.S. $\rightarrow$ heat in = heat out

$\hookrightarrow$ Energy absorbed = Energy lost

$$2000 \frac{W}{m^2} \times 0.80 = h\left(T_w - T_\infty\right) + \varepsilon\sigma\left(T_w^4 - T_\infty^4\right)$$

$$2000 \frac{W}{m^2} \times 0.80 = 15 \frac{W}{m^2\text{-}K}\left(T_w - \underset{+273\,K}{20°C}\right) + (0.50)\sigma\left(T_w^4 - \underset{+273\,K}{30°C^4}\right)$$

<must be Kelvin> <must be Kelvin>

$$\boxed{T_w = 377\ K}$$

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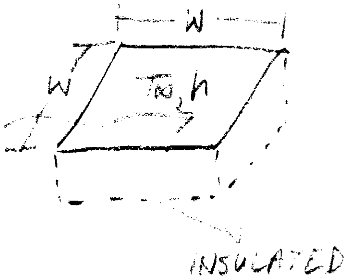
1.18) $W = 5mm$ $\quad h = 200 \frac{W}{m^2\text{-}K}$ air coolant, maximum allowable power?

$T_\infty = 15^\circ C$

$T_{max} = 85^\circ C$



$$\dot{q} = -h A_c \Delta T$$

$$= \left(-200 \frac{W}{m^2\text{-}K}\right)(0.005m \times 0.005m)^2 (15^\circ C - 85^\circ C)$$

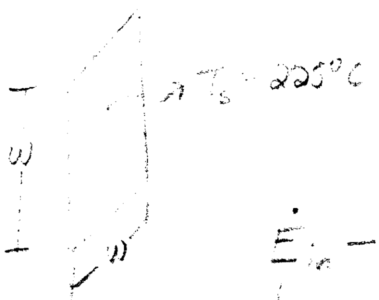
$$= \boxed{0.35\ W}$$

$h = 3000 \frac{W}{m^2\text{-}K}$ dielectric liquid

$$\dot{q} = -\left(3000 \frac{W}{m^2\text{-}K}\right)(0.005m \times 0.005m)^2 (15^\circ C - 85^\circ C)$$

$$= \boxed{5.25\ W}$$

1.22) Thin vert. plate suspended in still air



$\frac{dT}{dt} = -0.022\ K/s$ $\qquad W = 0.3\ m$

Ambient air $= 25^\circ C$ $\qquad m = 3.75\ kg$

$c_p = 2{,}770 \frac{J}{kg\cdot K}$

$$\underset{\downarrow\, 0}{\dot{E}_{in}} - \underset{\downarrow\, h}{\dot{E}_{out}} = \dot{E}_{change}$$

$$0 - hA(T_s - T_\infty) = m c_p \frac{dT}{dt} \longrightarrow \dot{E}_{in} = m c_p \frac{dT}{dt}$$

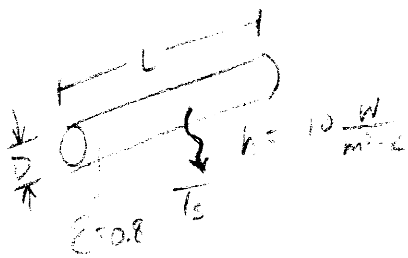
$$h = \frac{-m c_p \frac{dT}{dt}}{A(T_s - T_\infty)} = \frac{(-3.75kg)\left(2{,}770 \frac{J}{kg\cdot K}\right)(-0.022\ K/s)}{(0.3m \times 0.3m)(225^\circ C - 25^\circ C)}$$

$$= 12.7$$

1.22) Uninsulated steam pipe   surrounding walls & air at 25°C

$L = 25\,m$ pipe surface temperature of 150°C

$D = 100\,mm$ $h = 10\ \frac{W}{m^2\text{-}K}$, $\varepsilon = 0.8$



a) rate of heat loss?

$\dot{q}_{conv} = -hA\Delta T$

$= (-10\ \frac{W}{m^2\text{-}K})(\pi)(0.1\,m)(25\,m)(\ldots - \ldots°C)$

$= 9.8\ldots\ W$

view factor

$\dot{q}_{rad} = \varepsilon\sigma(T_s^4 - T_{sur}^4)F$

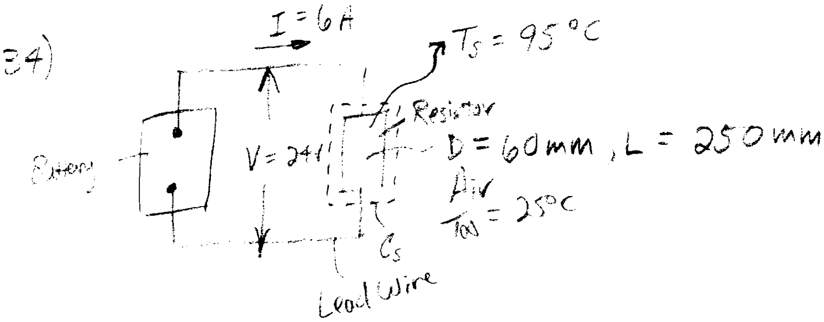
$= 0.8(\ldots)(423\,K^4 - 298\,K^4)(\pi)(0.1\,m)(25\,m)$

$=$

b) $\eta_f = 0.90$, $C_g = \$0.01/MJ$ annual cost of heat loss?

$E = 5.8 \times 10^{11}\ J = 18450 \times 3600 \times 24 \times 365$

1.34)



a) Find $\dot{E}_{in}$, $\dot{E}_g$, $\dot{E}_{out}$, $\dot{E}_{st}$

$$\dot{E}_{st} \equiv \frac{dE_{st}}{dt} = \dot{E}_{in} - \dot{E}_{out} + \dot{E}_g$$

$\dot{E}_{in} = 0$
$\dot{E}_{out} = 144\,W$
$\dot{E}_{st} = 0$
$\dot{E}_g = IV = 144\,W$

control surface of Resistor

b) volumetric heat generation rate?

=

=

$= 2.04 \times 10^5 \ \frac{W}{m^3}$

c) h?

$$\dot{E}_{out} = hA_{tot}(T_s - T_\infty)$$

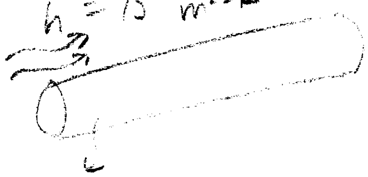
$$144\,W = h\left(\pi DL + \left(\frac{\pi}{4}D^2\right)\times 2\right)(95°C - 25°C)$$

$$h = 38.98 \ \frac{W}{m^2 \cdot K}$$

1.64)

$T_\infty = 25°C$

$h = 15 \frac{W}{m^2 \cdot K}$



$T_s = 200°C$

$D = 70mm$

$\varepsilon = 0.8$

a) $G = \sigma T_{sur}^4$

$$\frac{E_{net}}{L} = E_{rad} - \alpha G$$

$$= \pi D \left[\varepsilon \sigma (T_s^4) - \alpha \sigma T_{sur}^4 \right]$$

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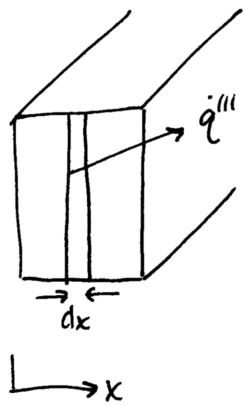
Heat of Diffusion Eq.



$$\dot{q}_{in} + \dot{q}_{gen} - \dot{q}_{out} = \dot{q}_{st}$$

$$\dot{q}_{in} = \dot{q}_x = -kA\frac{\partial T}{\partial x}$$

$$\dot{q}_{gen} = \dot{q}''' A\,dx \qquad \dot{q}''' = \left[\frac{W}{m^3}\right]$$

$$\dot{q}_{out} = -kA\frac{\partial T}{\partial x}\bigg|_{x+dx}$$

$$= -kA\left[\frac{\partial T}{\partial x} + \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)dx\right]$$

$$\dot{q}_{st} = mc\frac{\partial T}{\partial t}$$

$$\cancel{-kA\frac{\partial T}{\partial x}} + \dot{q}'''A\,dx - \cancel{kA\frac{\partial T}{\partial x}} + A\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) = \dot{q}_{st} = \rho c_p \frac{\partial T}{\partial t}A\,dx$$

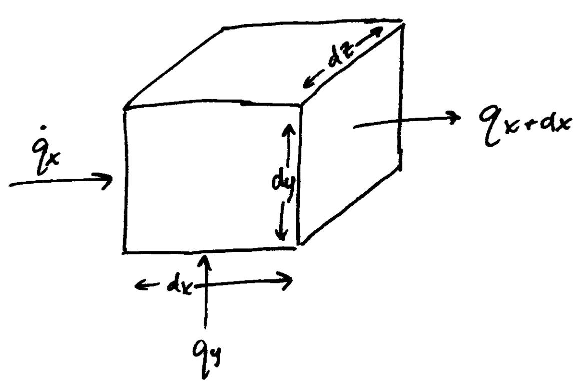
$$\dot{q}'''A\,dx + A\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) = \rho C_p\frac{\partial T}{\partial t}A\,dx$$

1-D

$$\boxed{\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \dot{q}''' = \rho c_p\frac{\partial T}{\partial t}}$$ - Heat of Diffusion Eq.

3-D

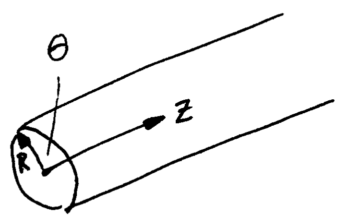
$$\frac{\partial}{\partial x}\left(K\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(K\frac{\partial T}{\partial z}\right) + \dot{q}''' = \rho c_p\frac{\partial}{\partial}$$



K = constant ⟨divide thru by K⟩

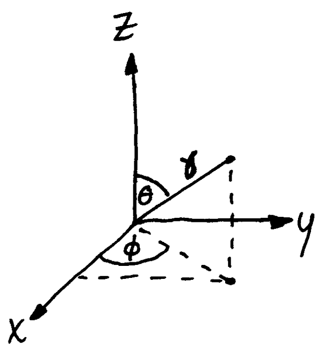
$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}'''}{K} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau} \qquad \alpha = \frac{K}{\rho c_p} = \text{thermal diff}$$

Cylindrical Coordinates



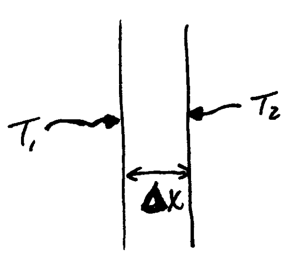
$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}'''}{K} = \frac{1}{r}\frac{\partial T}{\partial \tau}$$

Spherical Coordinates



$$\frac{1}{r}\frac{d^2}{dr^2}(rT) + \frac{1}{r^2 \sin\theta}\frac{d}{d\theta}\left(\sin\theta \frac{dT}{d\theta}\right) + \frac{1}{r^2 \sin^2\theta}\left(\frac{d^2T}{d\phi^2}\right) + \frac{\dot{q}'''}{k} = \frac{1}{r}\frac{dT}{d\tau}$$

Steady State 1-D Convection



$$\frac{\partial}{\partial x}\left(K\frac{\partial T}{\partial x}\right) = 0$$

$$K\frac{\partial T}{\partial x} = D$$

$$\frac{\partial T}{\partial x} = A \leftarrow D/K = A$$

integrate → $T = Ax + B$

Boundary Conditions:

$T = T_1$ @ $x = 0$

$T = T_2$ @ $x = \Delta x$

$T_1 = A \times 0 + B = B$

$T_2 = A\Delta x + B$

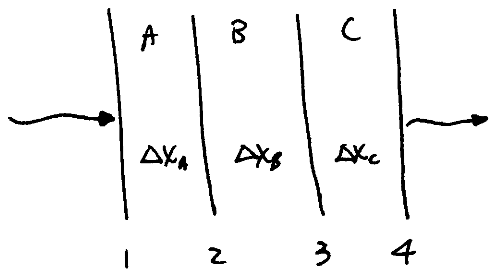
$$T = \frac{(T_2 - T_1)x}{\Delta x} + T_1$$

$$K = K_0(1 + BT)$$

- not a fcn. of x, y, z
- a function of T

$$q = -\frac{K_0 A}{\Delta x}\left[(T_2 - T_1) + \frac{B}{2}\left(T_2^2 - T_1^2\right)\right]$$

Composite Wall (s.s.)



$$q = -K_A A_A \frac{(T_2 - T_1)}{\Delta x_A} \qquad A_A = A_B = A_c$$

$$= -K_B A_B \frac{(T_3 - T_2)}{\Delta x_B}$$

$$= -K_c A_c \frac{(T_4 - T_3)}{\Delta x_c}$$

$$(T_1 - T_2) + (T_2 - T_3) + (T_3 - T_4) = \frac{q}{A}\left[\frac{\Delta x_A}{K_A} + \frac{\Delta x_B}{K_B} + \frac{\Delta x_c}{K_c}\right]$$

$$q = -\frac{K_{eff} A (T_4 - T_1)}{\Delta x_A + \Delta x_B + \Delta x_c}$$

$$K_{eff} = \frac{\Delta x_A + \Delta x_B + \Delta x_c}{\frac{\Delta x_A}{K_A} + \frac{\Delta x_B}{K_B} + \frac{\Delta x_c}{K_c}}$$

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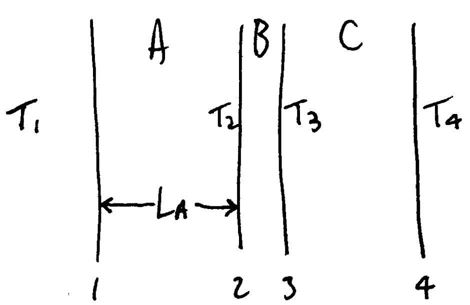


$$q_A = q_B = q_c$$

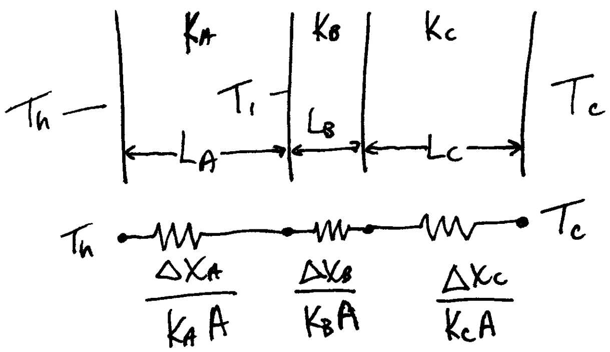
$$-K_A \frac{dT}{dx}\bigg|_A = -K_B \frac{dT}{dx}\bigg|_B = -K_c \frac{dT}{dx}\bigg|_c$$

Electrical Resistance Analogy

Electricity	Heat Transfer
I	$\dot{q}$
ΔV	ΔT
R	R_{th}

$$\dot{q} = -kA\frac{\Delta T}{\Delta x} \equiv I = \frac{V}{R}$$

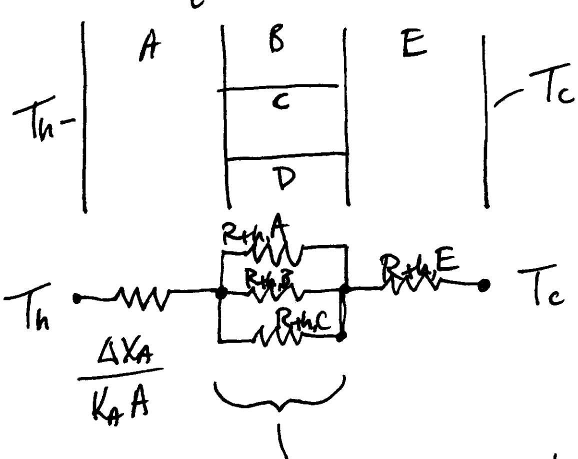
$$= \frac{\Delta T}{R_{th}} \longrightarrow \boxed{R_{th} = \frac{\Delta x}{kA}}$$



$$\dot{q} = \frac{T_h - T_c}{\sum R_{th}}$$

$$T_1: \quad \dot{q} = \frac{T_h - T_1}{\frac{\Delta X_A}{K_A A}}$$

$\dot{q} \longrightarrow$ 1-D

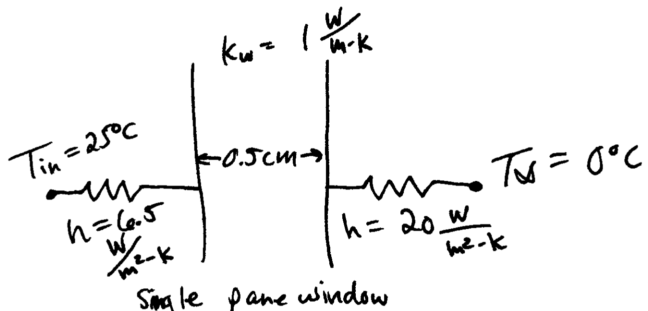


$$\frac{1}{R_{eq}} = \frac{1}{R_{th,A}} + \frac{1}{R_{th,B}} + \frac{1}{R_{th,C}}$$

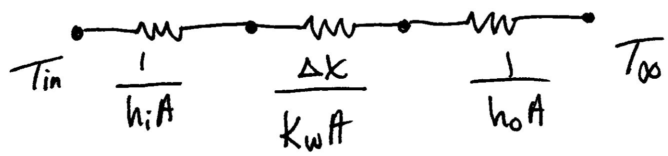
Convection Resistances

$$\dot{q} = hA(T_w - T_\infty)$$

$$= \frac{T_w - T_\infty}{\frac{1}{hA}} = \frac{T_w - T_\infty}{R_{th}} \longrightarrow R_{th} = \frac{1}{hA}$$



what is the heat flow per unit area?

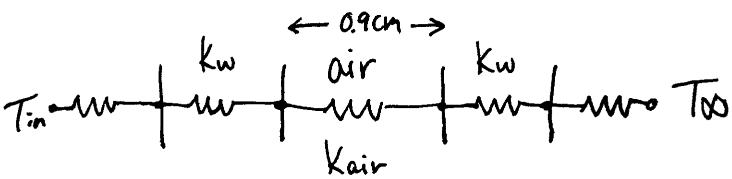


$$\dot{q} = \frac{T_i - T_\infty}{\frac{1}{h_i A} + \frac{\Delta x}{k_w A} + \frac{1}{h_o A}} \Rightarrow \frac{\dot{q}}{A} = \dot{q}'' = \frac{T_i - T_\infty}{\frac{1}{h_i} + \frac{\Delta x}{k_w} + \frac{1}{h_o}}$$

$$= \frac{25°C - 0°C}{\frac{1}{6.5 \frac{W}{m^2\text{-}K}} + \frac{0.005 m}{1 \frac{W}{m\text{-}K}} + \frac{1}{20 \frac{W}{m^2\text{-}K}}} = 120 \frac{W}{m^2}$$

---

double pane window:

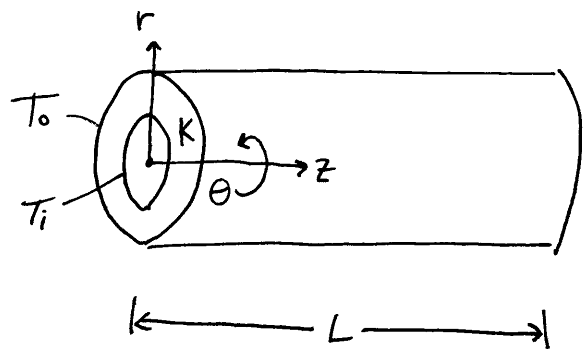


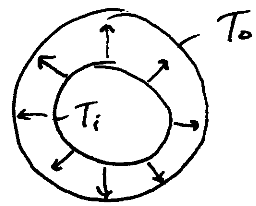
$$\dot{q}'' = \frac{\dot{q}}{A} = \frac{T_i - T_\infty}{\frac{1}{h_i} + \frac{\Delta x_w}{k_w} + \frac{\Delta x_{air}}{k_{air}} + \frac{\Delta x_w}{k_w} + \frac{1}{h_o}}$$

$$\dot{q}'' = 44 \frac{W}{m^2}$$

Cylindrical Systems

- consider a long hollow cylinder of length L, inner radius r_i, outer radius r_o





$$\dot{q} = -kA\frac{dT}{dr}$$

$$A(r) = 2\pi r L$$

$$\dot{q} = \frac{-k 2\pi \cancel{r} L}{\cancel{r}} \frac{T_o - T_i}{\ln(r_o/r_i)}$$

gov. eq: $$\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} + \frac{1}{r^2}\frac{d^2T}{d\theta^2} + \frac{d^2T}{dz^2} + \frac{\dot{q}'''}{k} = \frac{1}{\alpha}\frac{dT}{dt}$$

only dependent on radius: $$\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} = 0$$

$$\frac{d}{dr}\left(\frac{dT}{dr}\right) + \frac{1}{r}\frac{dT}{dr} = 0$$

$$\int \frac{d\left(\frac{dT}{dr}\right)}{\frac{dT}{dr}} = \int -\frac{1}{r}\,dr$$

$$\rightarrow \ln\left(\frac{dT}{dr}\right) = -\ln(r) + A \rightarrow \frac{dT}{dr} = C\frac{1}{r}$$

$$T = C\ln(r) + D$$

use B.C.: $T = T_i$ @ $r = r_i$, $T = T_o$ @ $r = r_o$

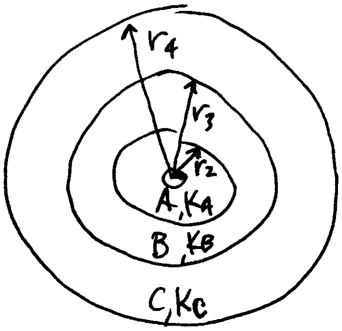
$$\boxed{\frac{T - T_i}{T_o - T_i} = \frac{\ln(r/r_i)}{\ln(r_o/r_i)}}$$

Temp. distribution in cylinder

$$\dot{q} = \frac{T_i - T_o}{\frac{\ln(r_o/r_i)}{2\pi k L}} = \frac{T_i - T_o}{R_{th}}$$

$$\boxed{R_{th} = \frac{\ln(r_o/r_i)}{2\pi k L}}$$

Multi-Layered Cylinders

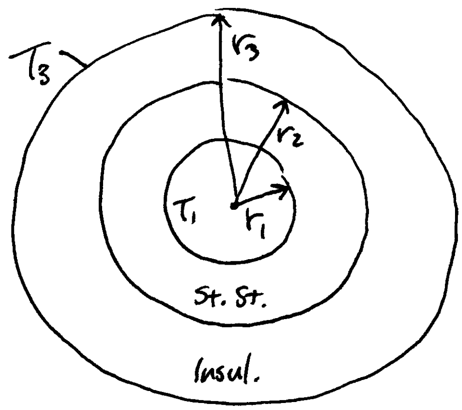


r_3, T_3
r_4, T_4
r_2, T_2
r_1, T_1

$$\boxed{R_{th} = \frac{\ln\left(\frac{r_o}{r_i}\right)}{2\pi K L}}$$

$$\dot{q} = \frac{\Delta T_{overall}}{R_{th,A} + R_{th,B} + R_{th,C}}$$

$$= \frac{(T_1 - T_4)\, 2\pi L}{\frac{\ln\left(\frac{r_2}{r_1}\right)}{K_A} + \frac{\ln\left(\frac{r_3}{r_2}\right)}{K_B} + \frac{\ln\left(\frac{r_4}{r_3}\right)}{K_C}}$$



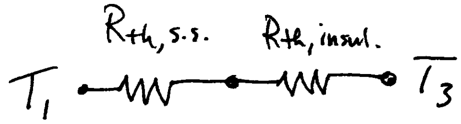
$T_1 = 600°C$

$T_3 = 100°C$

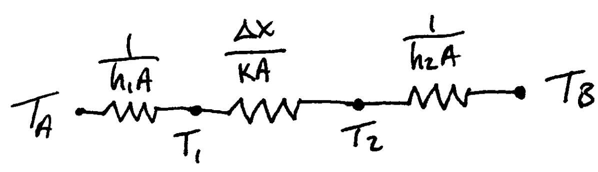
$r_1 = 0.01m$

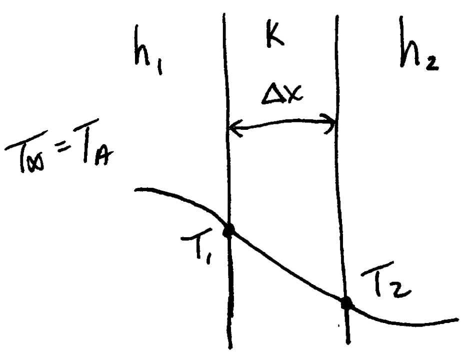
$r_2 = 0.02m$

$r_3 = 0.05m$



Overall H.T. Coeff.





$T_\infty = T_B$

$$\dot{q} = \frac{\Delta T}{\Sigma R_{th}} = \frac{T_A - T_B}{\frac{1}{h_1 A} + \frac{\Delta x}{KA} + \frac{1}{h_2 A}}$$

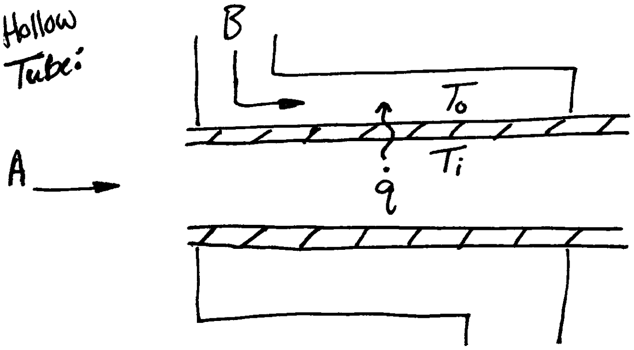
$$\rightarrow \dot{q} = UA\Delta T_{overall}$$

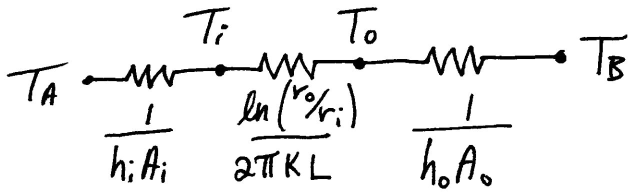
U - overall H.T. coeff.

$$\boxed{U = \frac{1}{\frac{1}{h_1} + \frac{\Delta x}{K} + \frac{1}{h_2}}}$$

Parallel Flow Heat Exchanger

Hollow Tube:





$$A_i = 2\pi r_i L \qquad A_o = 2\pi r_o L$$

$$\dot{q} = \frac{\Delta T_{overall}}{\Sigma R_{th}} = \frac{T_A - T_B}{\frac{1}{h_i A_i} + \frac{\ln(r_o/r_i)}{2\pi K L} + \frac{1}{h_o A_o}}$$

$$U_i = \frac{\dot{q}}{A_i (T_A - T_B)}$$

$$U_i = \frac{1}{\frac{1}{h_i} + \frac{A_i \ln(r_o/r_i)}{2\pi K L} + \frac{A_i}{h_o A_o}}$$

$$U_o = \frac{\dot{q}}{A_o (T_A - T_B)}$$

$$U_o = \frac{1}{\frac{A_o}{A_i h_i} + \frac{A_o \ln(r_o/r_i)}{2\pi K L} + \frac{1}{h_o}}$$

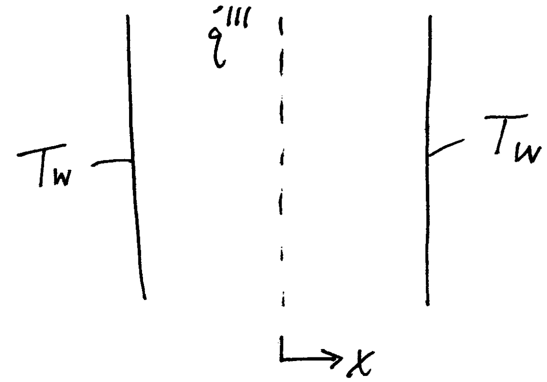
R Value for Insulation

$$R = \frac{\Delta T}{\dot{q}/A} = \frac{\Delta T}{\frac{KA\Delta T}{\Delta x\, A}} = \frac{\Delta x \uparrow}{K \downarrow} \qquad R\uparrow$$

- 6" Fiberglass $R = 3 \frac{W}{m^2 K}$

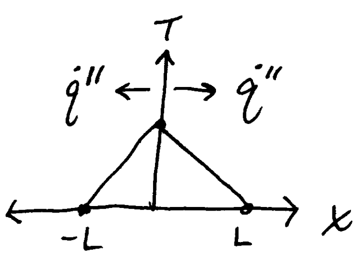
9/21/09

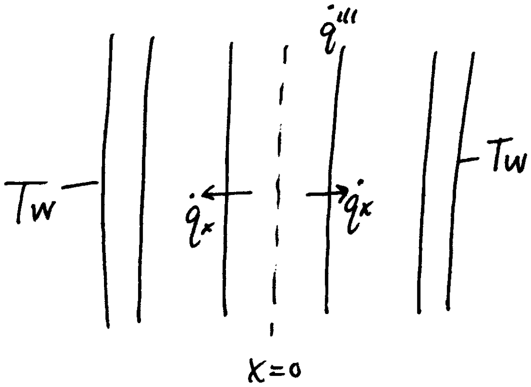
Internal Heat Generation (Cannot use resistance network analogy)

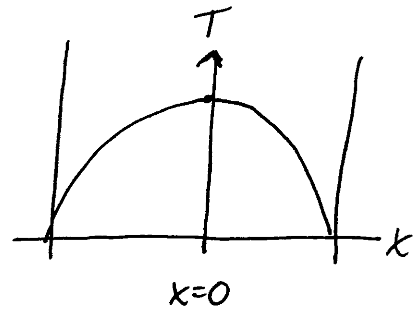


steady state

$\dot{q}''' =$ const.







Derive Temperature distribution in slab:

$$\frac{d}{dx}\left(k\frac{\partial T}{\partial x}\right) + \cancel{\frac{d}{dy}\left(k\frac{\partial T}{\partial y}\right)}^{\,0} + \cancel{\frac{d}{dz}\left(k\frac{\partial T}{\partial z}\right)}^{\,0} + \dot{q}''' = \cancel{\rho c_p \frac{dT}{dt}}^{\,0}$$

s.s.

$$\frac{d}{dx}\left(k\frac{\partial T}{\partial x}\right) = -\dot{q}''' \qquad k = \text{const.}$$

$$\frac{d}{dx}\left(\frac{\partial T}{\partial x}\right) = -\frac{\dot{q}'''}{k} \longrightarrow d\left(\frac{dT}{dx}\right) = -\frac{\dot{q}'''}{k}\,dx$$

$$\frac{dT}{dx} = -\frac{\dot{q}'''}{k}x + A$$

$$T = -\dot{q}'''\frac{x^2}{2k} + Ax + B$$

B.C. : $T = T_w$ @ $x = L$, $T = T_w$ @ $x = -L$

Solve for A, B to obtain

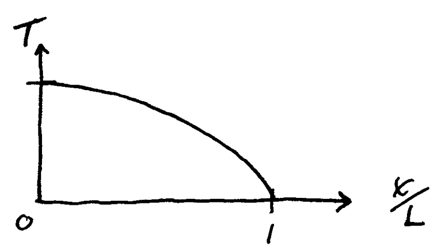
$$T - T_w = -\frac{\dot{q}'''}{2k}\left(x^2 - L^2\right)$$

T_{max} occurs @ $x = 0$: $T_{max} = T_w + \dfrac{\dot{q}''' L^2}{2k}$

or $T_{max} - T_w = +\dfrac{\dot{q}''' L^2}{2k}$

Non-dimensional temperature profile:

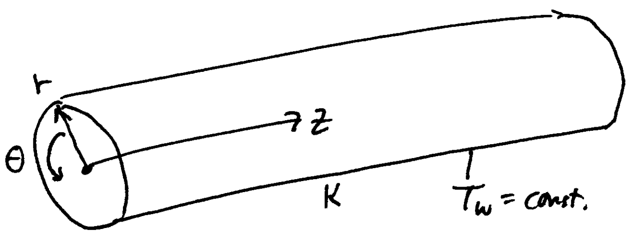
$$\frac{T - T_w}{T_{max} - T_w} = 1 - \frac{x^2}{L^2}$$



Cylindrical Systems

– solid cylinder, radius R, $\dot{q}''' = \text{const.}$, S.S.

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}'''}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$



symmetric axis, S.S., $\dot{q}''' = \text{const.}$

$$\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} = -\frac{\dot{q}'''}{K}$$

B.C.: $T = T_w$ @ $r = R$, $\frac{\partial T}{\partial r} = 0$ @ $r = 0$

Alternate B.C.: All energy generated in cylinder must be transferred by conduction at the wall.

$$\dot{q}''' \times V = \dot{q}''' \times \pi R^2 L = K A_s \left.\frac{dT}{dr}\right|_{r=R}$$

REWRITE

$$\dot{q}''' \pi R^2 L = K\, 2\pi R L \left.\frac{dT}{dr}\right|_{r=R}$$

$$\dot{q}''' R = 2K \left.\frac{dT}{dr}\right|_{r=R} \longleftarrow \text{Alt. B.C.}$$

$$\cancel{\text{then}\ \frac{d}{dr}\left(r\frac{dT}{dr}\right) = \frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr}}$$

$$\underbrace{r\frac{d^2T}{dr^2} + \frac{dT}{dr}}_{} = -\frac{\dot{q}''' r}{K}$$

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = \frac{-\dot{q}''' r}{K} \xrightarrow{\text{INTEGRATE}} r\frac{dT}{dr} = -\frac{\dot{q}''' r^2}{2K} + C$$

Apply B.C.: $\frac{dT}{dr} = 0$ @ $r = 0 \rightarrow C = 0$

$$\frac{dT}{dr} = -\frac{\dot{q}'''r}{2K} \longrightarrow T = \frac{-\dot{q}'''r^2}{4K} + C_2$$

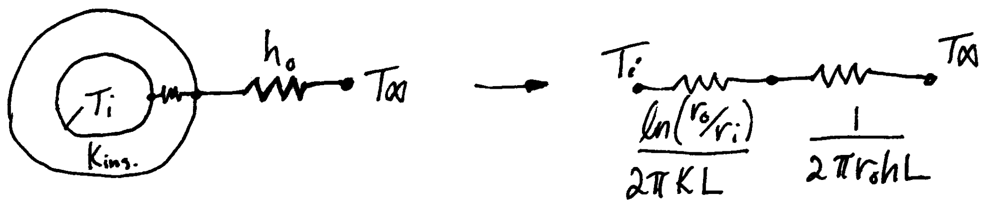
B.C.: $T = T_w$ @ $r = R \longrightarrow C_2 = T_w + \frac{\dot{q}'''R^2}{4K}$

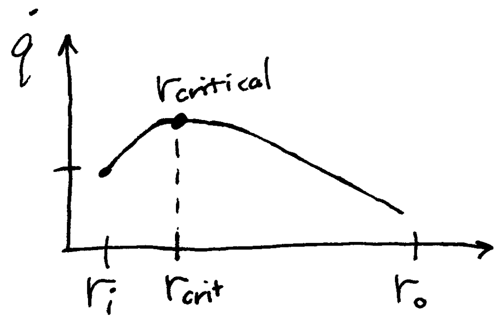
$$T - T_w = \frac{\dot{q}'''}{4K}\left(R^2 - r^2\right)$$

@ $r=0$ $\quad T_0 = \frac{\dot{q}'''R^2}{4K} + T_w$ (max Temp. in cylinder)

discussion 9/22/09

Critical Radius of Insulation





$$\dot{q} = \frac{T_i - T_\infty}{\frac{\ln(r_o/r_i)}{2\pi KL} + \frac{1}{2\pi L h r_o}}$$

To find r_{crit}: $\frac{d\dot{q}}{dr_o} = 0$ $\qquad \boxed{r_{crit} = \frac{K}{h}}$

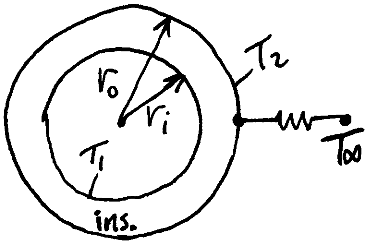
$$= \frac{(T_i - T_\infty)\,2\pi L K h}{h\ln(r_o/r_i) + K/r_o} = \frac{(T_i - T_\infty)\,2\pi L K h}{h(\ln r_o - \ln r_i) + K/r_o} = \frac{(T_i - T_\infty)\,2\pi L K h\, r_o}{h\ln(r_i) r_o}$$

$\ln r_o - \ln r_i$

$$\frac{d\dot{q}}{dr_o} = \frac{d}{dr_o}\left((T_i - T_\infty)\left(\frac{\ln(r_o/r_i)}{2\pi KL} + \frac{1}{2\pi L h r_o}\right)^{-1}\right)$$

$= (T_\infty - T_i)\Big($

ex)



$T_1 = 200°C \qquad h = 3 \frac{W}{m^2 \cdot K} \qquad r_i = 2.5\,cm$

$T_\infty = 20°C \qquad K_{ins} = 0.17 \frac{W}{m \cdot K}$

- what is $\dot{q}$ w/o insulation?

$\cancel{\dot{q} = K_{ins} A_s}$ $\qquad \dot{q} = h A_s (T_1 - T_\infty)$

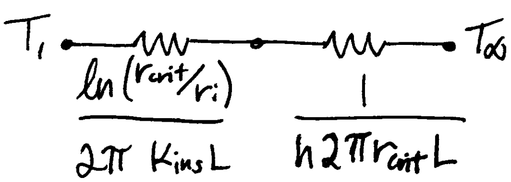
$$\frac{\dot{q}}{L} = 3 \frac{W}{m^2 \cdot K} \times 2\pi (0.025m)(200°C - 20°C)$$

$$= 84.8 \, W/m$$

- what is $\dot{q}$ w/ r_{crit} of insulation

$$r_{crit} = \frac{k_{ins}}{h} = \frac{0.17 \frac{W}{m \cdot K}}{3 \frac{W}{m^2 \cdot K}} = 0.0567\ m$$

$$\cancel{\frac{\dot{q}}{L} = k_{ins} A_s \frac{(T_1 - T_2)}{r_{crit} - r_i} = 0.17 \frac{W}{m \cdot K} \times 2\pi(\quad) \frac{(200°C - T_2)}{(0.0567 - 0.025)m}}$$

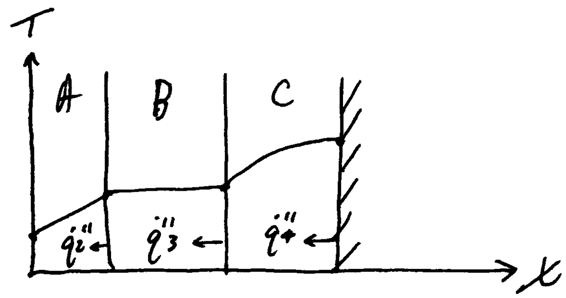


$$\frac{\dot{q}}{L} = \frac{T_1 - T_\infty}{\Sigma R_{th}} = 106 \, W/m$$

Internal Heat Generation

$$\dot{q} = KA\frac{\Delta T}{\Delta x} \rightarrow \Delta T(\Delta x) = \frac{\dot{q}}{KA}\Delta x$$

S.S. heat generation in a plane wall of three different materials, each w/ constant K.



a) comment on the relative magnitudes of $\dot{q}_2''$, $\dot{q}_3''$, & $\dot{q}_4''$

b) comment on relative magnitudes of $K_A, K_B,$ & K_C

c) sketch $\dot{q}''(x)$

$$T(x) = \frac{\dot{q}}{K\cancel{A}}x \qquad \dot{q}'' = K\frac{\Delta T}{\Delta x} \rightarrow T(x) = \frac{\dot{q}''}{K}x$$

a) $\dot{q}_4'' = 0$

Inside C: $\dot{q}''' \neq 0 \rightarrow$ internal heat generation

The curve in C indicates internal heat generation

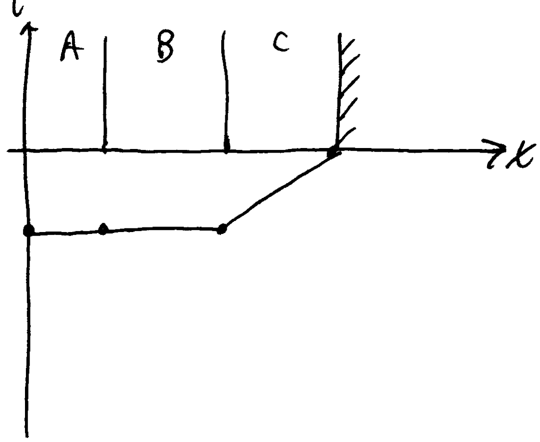
$\dot{q}_2'' = \dot{q}_3''$

b) $K_A < K_B \qquad K_B > K_C$

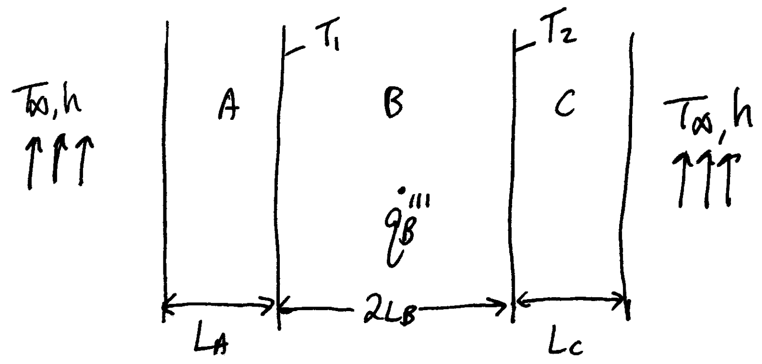
$\dot{q}'' = \dot{q}_2'' = \dot{q}_3''$

$K_A\frac{dT}{dx_A} = K_B\frac{dT}{dx_B}$

c) $\dot{q}_x''$



3.73)



$T_\infty = 25°C$

$h = 1000 \frac{W}{m^2\text{-}K}$

internal heat generat. $\dot{q}_B'''$

$T_1 = 261°C$, $T_2 = 211°C$

$K_A = 25 \frac{W}{m\text{-}K}$ $\quad L_A = 30mm$

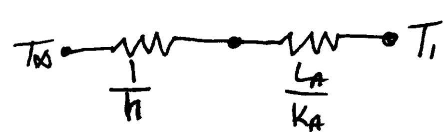
$K_C = 50 \frac{W}{m\text{-}K}$ $\quad L_B = 30\ mm$

$L_C = 20mm$

a) assuming negligible contact resistance at the interfaces, determine $\dot{q}_B'''$ & K_B

$$\dot{q}''' = K \frac{1}{L}\frac{dT}{dx} = \frac{\dot{q}''}{L} \longrightarrow \dot{q}'' = \dot{q}''' \times L \longrightarrow \boxed{\dot{q}_B''' L_B = \dot{q}_1'' + \dot{q}_2''}$$

$$\cancel{\dot{q}_A'' = K_A \frac{T_1 - T_\infty}{L_A} = 25 \frac{(261-25)}{0.03} = 197 \frac{kW}{m^2}}$$



$$\dot{q}_1'' = \frac{T_1 - T_\infty}{\Sigma R_{th}} = \frac{534K - 298K}{\frac{1}{1000\frac{W}{m^2\text{-}K}} + \frac{0.03\,m}{25\frac{W}{m\text{-}K}}} = 107 \frac{kW}{m^2}$$

$$\dot{q}_2'' = \frac{484K - 298K}{\frac{0.02m}{50\frac{W}{m\text{-}K}} + \frac{1}{1000\frac{W}{m^2\text{-}K}}} = 133 \frac{kW}{m^2}$$

$$\dot{q}_B''' = \frac{\dot{q}_1'' + \dot{q}_2''}{2L_B} = \frac{107{,}000 + 133{,}000}{2(0.03)} = \boxed{4 \frac{MW}{m^3}}$$

$$\cancel{2L_B \dot{q}_B''' = K_B \frac{dT}{dx} \rightarrow K_B =}$$

$$T(x) = \frac{-\dot{q}'''x^2}{2K_B} + C_1 x + C_2$$

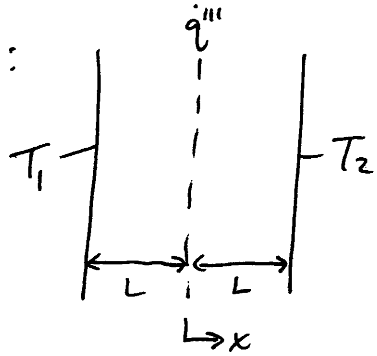
$$\left[\frac{\partial T(x)}{\partial x}\right] = \left[\frac{-\dot{q}'''x}{K_B} + C_1\right] K_B = \dot{q}'''x - C_1 K_B$$

9/23/09

Recap: Int. Heat Generation

S.S., $\dot{q}''' = \text{const.}$, $k = \text{const.}$

Planar Systems:



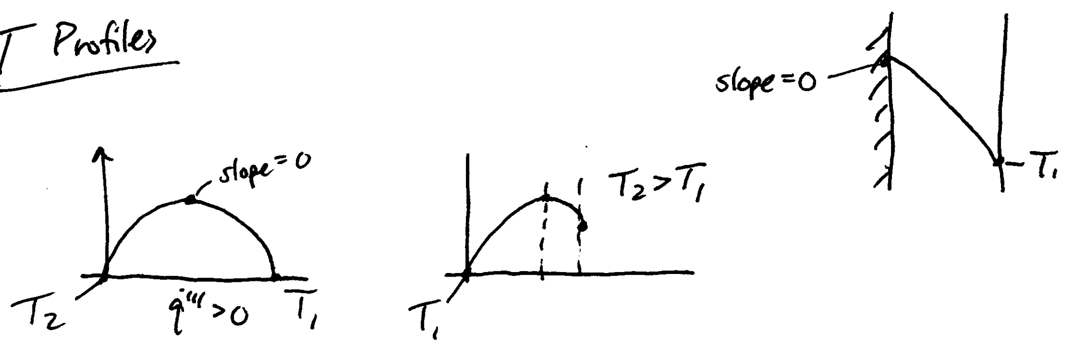
$$\boxed{\begin{gathered}\frac{d^2T}{dx^2} + \frac{\dot{q}'''}{k} = 0 \\ T = \frac{-\dot{q}''' x^2}{2k} + C_1 x + C_2 \\ C_1, C_2 \text{ obtained from B.C.}\end{gathered}}$$

$$T(x) = \frac{\dot{q}''' L^2}{2k}\left(1 - \frac{x^2}{L^2}\right) + \frac{T_2 - T_1}{2}\frac{x}{L} + \frac{T_1 + T_2}{2}$$

$$T_1 = T_2 = T$$

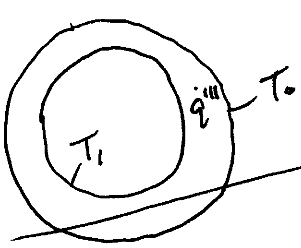
$$T(x) = \frac{\dot{q}''' L^2}{2k}\left(1 - \frac{x^2}{L^2}\right) + T$$

T Profiles



~~Radial Systems~~

$$\cancel{T = T_o + \frac{\dot{q}'''}{4k}(r_o^2 - r_i^2) + C\frac{r}{r_o}, \quad C = \frac{T_i - T_o + \dot{q}'''(r_i^2 - r_o^2)/4k}{\ln(r_o/r_i)}}$$



Cylinders w/ Heat Generation

- Uniformly distributed heat sources
- constant conductivity

$$\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} + \frac{\dot{q}'''}{k} = 0 \qquad \text{— gov. eq.}$$

B.C.: $T = T_w$ @ $r = R$, $\frac{dT}{dr} = 0$ @ $r = 0$

Alt. B.C.: Heat flux at surface = energy generated within the solid

$$\dot{q}'''\pi R^2 L = -k\, 2\pi R L \left.\frac{dT}{dr}\right|_{r=R}$$

$$r\frac{d^2T}{dr^2} + \frac{dT}{dr} = -\frac{\dot{q}'''r}{k}$$

-or- $$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = \frac{-\dot{q}'''r}{k} \xrightarrow{\text{integrate}} r\frac{dT}{dr} = \frac{-\dot{q}'''r^2}{2k} + A$$

B.C.: $0 = \frac{dT}{dr}$ @ $r = 0$ $\rightarrow A = 0$

$$\xrightarrow{\text{integrate}} T = \frac{-\dot{q}'''r^2}{4k} + B$$

B.C.: At $r = R$, $T = T_w$

$$\boxed{T - T_w = \frac{\dot{q}'''}{4k}(R^2 - r^2)}$$

Non-dimensional:

$$\boxed{\frac{T - T_w}{T_0 - T_w} = 1 - \left(\frac{r}{R}\right)^2}$$

$T_0 = T(r=0)$

$$T_0 = \frac{\dot{q}'''R^2}{4k} + T_w$$

Hollow Cylinder

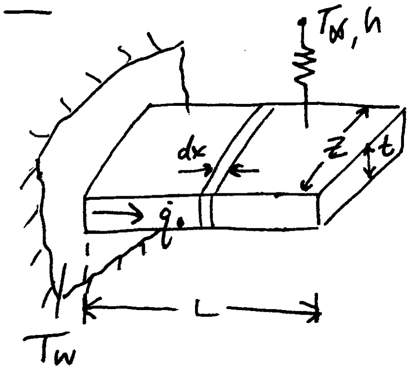
B.C.: $T = T_i$ @ $r = r_i$, $T = T_0$ @ $r = r_0$

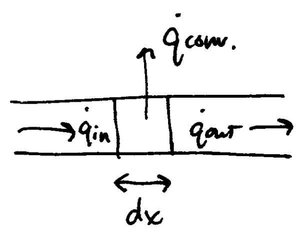
$$\boxed{T - T_0 = \frac{\dot{q}'''}{4k}(r_0^2 - r^2) + A\ln\left(r/r_0\right)}$$

$$\boxed{A = \frac{T_i - T_0 + \dot{q}'''(r_i^2 - r_0^2)/4k}{\ln\left(r_i/r_0\right)}}$$

9/28/09

Fins





Energy Balance: $\dot{q}_{in} = \dot{q}_{out} + \dot{q}_{conv}$

$$\begin{cases} \dot{q}_{in}(x) = -kA\frac{dT}{dx} = -k(zt)\frac{dT}{dx} \\ \dot{q}_{out}(x) = \dot{q}_{in} + \frac{d}{dx}\left(\dot{q}(x)\right)dx \\ \dot{q}_{conv} = hA_s\Delta T = h\underbrace{(2z+2t)}_{\text{perim.}}dx\,(T(x) - T_\infty) \end{cases}$$

– Taylor Expansion

E. Balance: $\cancel{\dot{q}_{in}(x)} = \cancel{\dot{q}_{in}} + \frac{d}{dx}(\dot{q}(x))dx + hP\,dx\,(T(x) - T_\infty)$

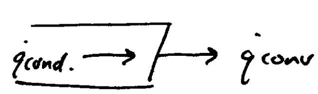
solve to get: $\boxed{\frac{d^2T}{dx^2} - \frac{hP}{kA}(T - T_\infty) = 0}$

Some B.C.

1) Fin is infinitely long: T at tip $= T_\infty$

2) Fin tip is insulated: $\frac{dT}{dx} = 0$ @ $x = L$

3) Fin loses heat by convection at tip:



$\dot{q}_{cond.} = \dot{q}_{conv.}$

$$-kA\frac{dT}{dx}\bigg|_{x=L} = hA(T_L - T_\infty)$$

General Solution to Gov. Eq :

Define $\theta = T - T_\infty \rightarrow \frac{d\theta}{dx} = \frac{dT}{dx}, \quad \frac{d^2\theta}{dx^2} = \frac{d^2T}{dx^2}$

$$\rightarrow \frac{d^2\theta}{dx^2} - \frac{hP}{KA}\theta = 0$$

Define $m^2 = \frac{hP}{KA} \longrightarrow \boxed{\frac{d^2\theta}{dx^2} - m^2\theta = 0}$

Try $\theta = \sin(mx), \quad \frac{d\theta}{dx} = m\cos(mx), \quad \frac{d^2\theta}{dx^2} = -m^2\sin(mx)$

Try $\theta = e^{mx}, \quad \frac{d\theta}{dx} = me^{mx}, \quad \frac{d^2\theta}{dx^2} = m^2e^{mx}$

gen. sol. : $\boxed{\theta = C_1e^{mx} + C_2e^{mx}}$

Case 1) Assume fin is infinitely long : $T \rightarrow T_\infty$ @ $x \rightarrow \infty$

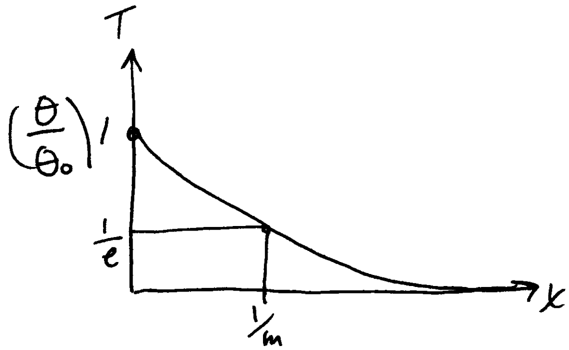
$$T = T_\infty \rightarrow \theta = T - T_\infty = 0$$

$$0 = C_1e^{m\infty} + \cancel{C_2e^{-m\infty}}^{\rightarrow 0} \qquad C_1 = 0$$

@ $x = 0, \ T = T_w \rightarrow \theta_0 = T_w - T_\infty$

$$\theta_0 = C_2e^{-m \times 0} \rightarrow C_2 = \theta_0$$

$\rightarrow \theta = \theta_0e^{-mx}$ or $\boxed{\frac{\theta}{\theta_0} = e^{-mx}}$ non-dimensional



Case 2) Tip is insulated : $\left.\frac{dT}{dx}\right|_{x=L} = 0$, $T = T_\omega$ @ $x = 0$

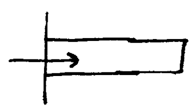
- solve for C_1, C_2

$$\boxed{\frac{\theta}{\theta_0}} = \frac{e^{-mx}}{1+e^{-2mL}} + \frac{e^{mx}}{1+e^{2mL}} = \boxed{\frac{\cosh(m(L-x))}{\cosh(mL)}}$$

Case 3) convection at tip : $\left. KA\frac{dT}{dx}\right|_{x=L} = hA(T-T_\infty)$

$$\frac{T-T_\omega}{T_\omega - T_\infty} = \boxed{\frac{\theta}{\theta_0} = \frac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + \frac{h}{mk}\sinh mL}}$$

$$\dot{q}_0 = -KA\left.\frac{dT}{dx}\right|_{x=0}$$

$\dot{q}_0$ 

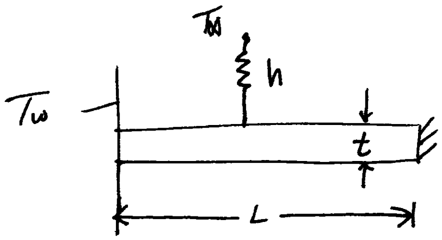
To find heat flux : compute $\dot{q}_0 = -KA\left.\frac{dT}{dx}\right|_{x=0}$

B.C. 1 : $\dot{q}_0 = \theta_0\sqrt{hPKA}$

B.C. 2 : $\dot{q}_0 = \sqrt{hPKA}\,\theta_0\tanh(mL)$

B.C. 3 : $\dot{q}_0 = \sqrt{hPKA}\,(T_\omega - T_\infty)\dfrac{\sinh(mL) + h/mk\cosh(mL)}{\cosh(mL) + \frac{h}{mk}\sinh(mL)}$

ex) Aluminum Fin: $K = 200 \frac{W}{m\text{-}K}$, 3 mm thick, 7.65 mm long, rect. c.s.
infinitely ~~thin~~ deep, $T_\infty = 50°C$, $h = 10 \frac{W}{m^2\text{-}K}$, $T_w = 300°C$



– case 2

$$z \to \infty, \quad 2t \to 0$$

$$m = \sqrt{\frac{hP}{KA}} = \sqrt{\frac{h(2z + \cancel{2t})}{k(zt)}} = \sqrt{\frac{h(2z)}{k(zt)}} = \sqrt{\frac{2h}{kt}} = 5.77 \ \frac{1}{m}$$

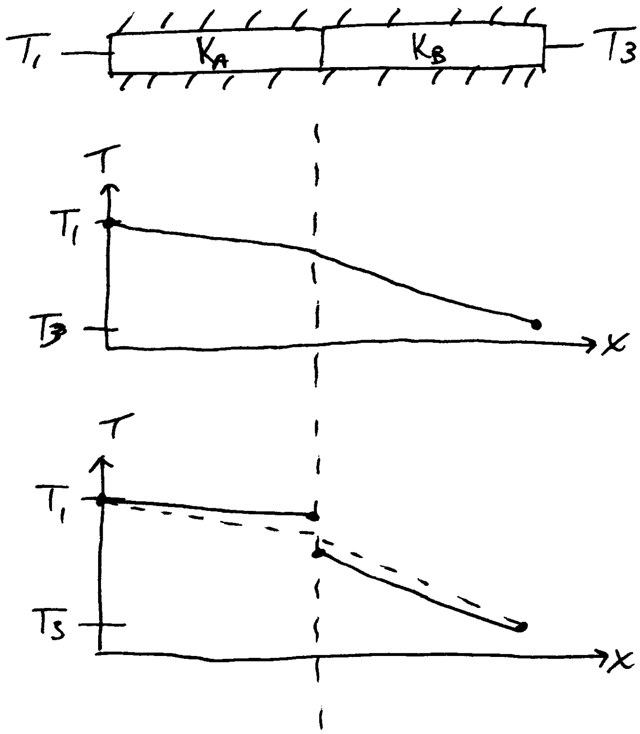
$$\dot{q}_o = \tanh(mL)\sqrt{hPKA}\,\theta_o \qquad hPKA = h(2z + \cancel{2t})k(zt) = 2hkz^2t$$

$$\frac{\dot{q}_o}{z} = \dot{q}_o' = \tanh(mL)\sqrt{2hkt}\,\theta_o \longrightarrow \underline{\underline{\dot{q}_o' = 360 \ \frac{W}{m}}}$$

discussion 9/29/0

Sect. 3.1.4 – contact resistance

two rods in contact with each other

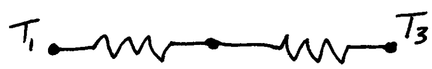


$K_A > K_B$

$T_1 > T_3$

$\dot{q}''' = 0$, S.S.

– Ideal temp. profile



– Actual temp. profile

– Jump in temp. due to contact resistance.

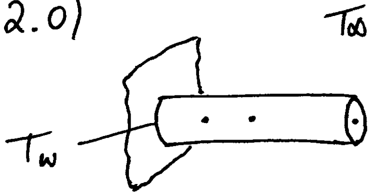
$$\dot{q}'' = K_A \frac{T_1 - T_{2A}}{\Delta X_A} = K_B \frac{T_{2B} - T_3}{\Delta X_B} = \frac{T_{2A} - T_{2B}}{1/h_c}$$

$\frac{1}{h_c}$ = contact resistance

$\frac{1}{h_c} = R''_{t,c}$ (book)

$\left[\frac{m^2\text{-}K}{W}\right]$

3.1.2.0)



$L = 100\ mm$ $\quad d = 5\ mm$ $\quad K = 133\ \frac{W}{m\text{-}K}$

$T_w = 200°C$ $\quad T_\infty = 20°C$ $\quad h = 30\ \frac{W}{m^2\text{-}K}$

$T_{25mm} = ?$ $\quad T_{50mm} = ?$ $\quad T_{100mm} = ?$

$$m = \sqrt{\frac{hP}{KA_c}} = \sqrt{\frac{h\pi d}{K\frac{\pi}{4}d^2}} = 13.43\ m^{-1}$$

$$\frac{h}{mK} = 0.0168$$

$$\frac{\theta}{\theta_o} = \frac{\cosh m(L-x) + \left(\frac{h}{mK}\right)\sinh m(L-x)}{\cosh mL + \frac{h}{mK}\sinh mL}$$

$$\theta = T - T_\infty$$
$$\theta_o = T_w - T_\infty$$

$$T(x) = \frac{\cosh m(L-x) + \left(\frac{h}{mK}\right)\sinh m(L-x)}{\cosh mL + \frac{h}{mK}\sinh mL}\left(T_w - T_\infty\right) + T_\infty$$

3.124) Two long copper rods soldered together

$d = 10mm$

Melting $Pt_{solder} = 650°C$

$T_\infty = 25°C$

$h = 10 \frac{W}{m^2\text{-}K}$

$K = 379 \frac{W}{m\text{-}K}$

$P_{min} = ?$

T_∞

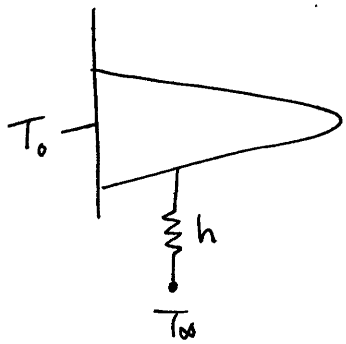
heat flux: $q_f = \sqrt{hPKA_c}\,(T - T_\infty) \times 2$ rods

$$= 2\sqrt{h\pi d K \frac{\pi}{4} d^2}\,(T - T_\infty) = 120.9 \text{ W}$$

9/30/09

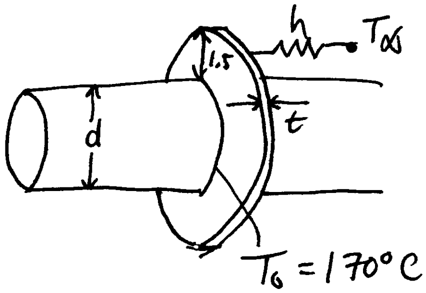
Fin Efficiency:

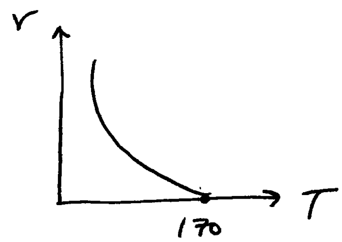
$$\eta_{fin} = \frac{\text{actual heat transfer}}{\text{maximum possible H.T. by fin}} = \frac{\dot{q}_f}{hA_f(T_o - T_\infty)}$$



η values – Fig 3.18, 3.19, Table 3.5

ex) Aluminum fin 1.5 cm wide, 1.0 mm thick, on a 2.5 cm diameter tube. Tube surface at 170°C, $T_\infty = 25°C$, $h = 130\ \frac{W}{m^2\text{-}K}$, $K_{Al} = 200\ \frac{W}{m\text{-}K}$. H.T. per fin?





$$\boxed{\dot{q}_{max} = hA_f(T_o - T_\infty)} = \left(130\ \frac{W}{m^2\text{-}K}\right)\left[2\pi(r_o^2 - r_i^2) + 2\pi r_o t\right](170°C - 25°C)$$

To find η_f, ~~table~~ fig. 3.19

$$r_{2c} = r_2 + \frac{t}{2} = r_o + \frac{t}{2}$$

$$L_c = L + \frac{t}{2} = (r_o - r_i) + \frac{t}{2}$$

$$L_c^{3/2}\left(\frac{h}{kA_p}\right)^{1/2} \quad A_p = L_c t$$

$$L_c = L + \frac{t}{2} = 1.5\,cm + \frac{1\,mm}{2} = 0.0155\,m$$

$$r_{2c} = r_1 + L_c = 1.25\,cm + 1.55\,cm = 0.0280\,m$$

$$A_p = t(r_{2c} - r_1) = (1\times10^{-3})(0.028 - 0.0125) = 1.55\times10^{-5}\,m^2$$

$$L_c^{3/2}\left(\frac{h}{kA_p}\right) = 0.395$$

$$\frac{r_{2c}}{r_1} = 2.24$$

From 3.19, $\eta_f = 0.82$

$$\eta_{effectiveness} = \frac{\text{heat transfer w/ fin}}{\text{heat transfer w/o fin}} = \frac{\eta_f A_f h (T_0 - T_\infty)}{h A_b (T_0 - T_\infty)}$$

Chapter 4

10/5/09

2-D Conduction

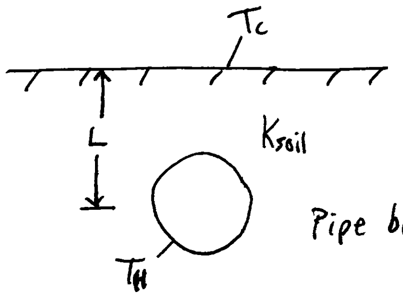
Conduction shape factor:

Applicable when the temperature of the body & surroundings are specified

$$\boxed{\dot{q} = k S \Delta T_{overall}}$$

k – thermal conductivity of medium

S – conduction shape factor, Table 4-1



Pipe buried in ground

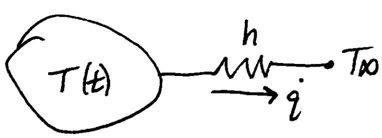
Unsteady Heat Conduction

Temperatures can change w/ time

- Lumped capacity (Uniform temperature through-out, but changing w/ time)
- semi-infinite solid
- multi-dimensional conduction

Lumped Capacity

- Temperature in body is spatially uniform, but changing w/ time.



Heat loss by convection results in temp drop in body.

$$\dot{q} = h A_s (T(t) - T_\infty) = -m c_p \frac{dT}{dt} = -\rho V c_p \frac{dT}{dt}$$

B.C.: $T = T_i$ @ $t = 0$

Separate Variables:

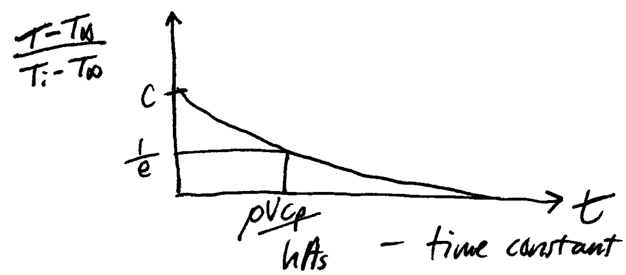
$$\frac{dT}{T - T_\infty} = -\frac{hA_s}{mc_p} dt$$

$$\ln(T - T_\infty) = -\frac{hA_s}{\rho V c_p} t + C_1$$

$$T - T_\infty = \exp\left(-\frac{hA_s}{\rho V c_p} t\right) C_2$$

I.C.: $T = T_i$ @ $t = 0 \longrightarrow C_2 = T_i - T_\infty$

$$\boxed{\frac{T - T_\infty}{T_i - T_\infty} = \exp\left[-\frac{hA_s}{\rho V c_p} t\right]}$$



- If conduction in body ~~is~~ large, R_{th} in body is small and can be neglected, so Temp is uniform

$$q'' \sim k \frac{T_i - T_o}{L} \sim h(T_o - T_\infty)$$

$$\rightarrow T_i - T_o \sim \frac{hL}{k}(T_o - T_\infty + T_i - T_i)$$

$$\sim B_i(T_o - T_i) + B_i(T_i - T_\infty)$$

$$\rightarrow \frac{T_i - T_o}{T_i - T_\infty} = \frac{B_i}{1 + B_i}$$ - For lumped capacity, $B_i \ll 1$

- For most geometries, use $L = \frac{V}{A_s}$. If $B < 0.1$, solutions will be within 5%

Example

Steel ball bearing: $c_p = 0.46 \frac{kJ}{\quad}$, $k = 35 \frac{W}{m\text{-}K}$, $D = 5\,mm$, $T_i = 450°C$, $h = 10 \frac{W}{m^2\text{-}K}$, $T_\infty = 100°$

How long until $T = 150°C$?

$$B_i = \frac{h(V/A_s)}{k} = \frac{h\left(\frac{4}{3}\pi r^3 / 4\pi r^2\right)}{k} = \frac{h(r/3)}{k} \longrightarrow B_i = 0.0024 < 0.1 \quad \therefore \text{lumped capacity}$$

$$\frac{T - T_\infty}{T_i - T_\infty} = \exp\left(\frac{-hA_s}{\rho c_p V} t\right), \quad \frac{hA_s}{\rho V c_p} = 3.34 \times 10^{-4} s^{-1}$$

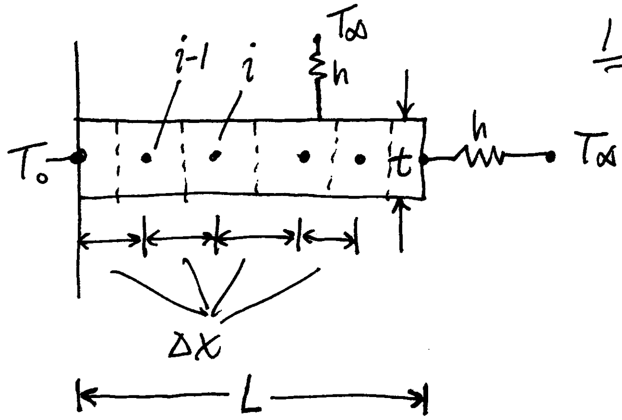
$$\frac{T - T_\infty}{T_i - T_\infty} = 0.143 = \exp(-3.34E{-4}\, t)$$

$$\boxed{t = 5826\ s}$$

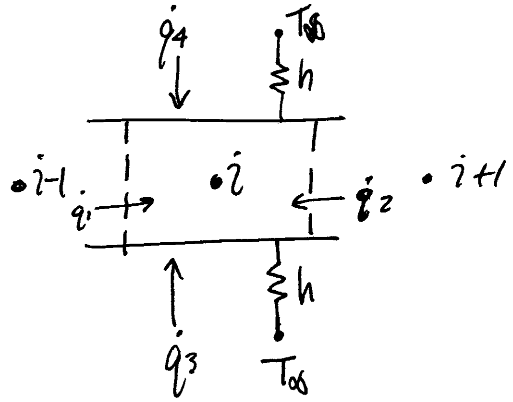
discussion 10/6/09

Numerical Method for Heat Conduction

1-D Fin



1) Assign nodes on fin
2) Draw control forces around each node that are equally spaced between nodes
3) Perform energy balance on each node



Energy balance on node i: $\dot{q}_1 + \dot{q}_2 + \dot{q}_3 + \dot{q}_4 = 0$

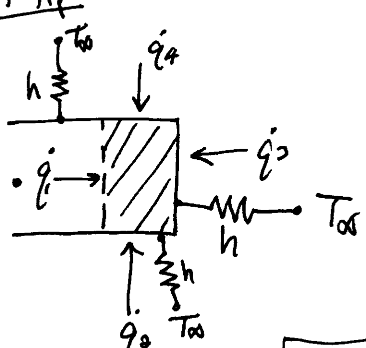
$$\underbrace{kt\frac{T_{i-1}-T_i}{\Delta x}}_{\dot{q}_1} + \underbrace{kt\frac{T_{i+1}-T_i}{\Delta x}}_{\dot{q}_2} + \underbrace{2h\Delta x(T_\infty - T_i)}_{\dot{q}_3+\dot{q}_4} = 0$$

Solve for T_i: $T_i\left[2\frac{kt}{\Delta x} + 2h\Delta x\right] = \frac{kt}{\Delta x}(T_{i-1}+T_{i+1}) + 2h\Delta x T_\infty$

$$\rightarrow \boxed{T_i = \frac{\frac{kt}{\Delta x}(T_{i-1}+T_{i+1}) + 2h\Delta x T_\infty}{2\frac{kt}{\Delta x} + 2h\Delta x}}$$

node eq. for node i

At tip:



$$\Sigma \dot{q}_i = 0$$

$$kt\frac{T_{n-1}-T_n}{\Delta x} + h\frac{\Delta x}{2}(T_\infty - T_n) + h\frac{\Delta x}{2}(T_\infty - T_n) + ht(T_\infty - T_n) = 0$$

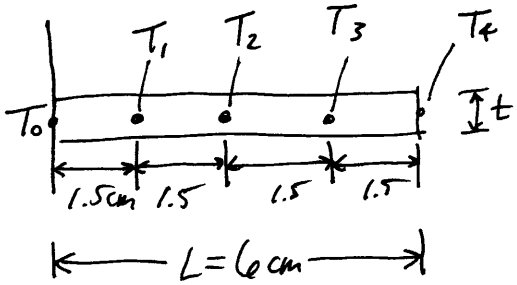
Solve for T_n:

$$\boxed{T_n = \frac{\frac{kt}{\Delta x}T_{n-1} + htT_\infty + h\Delta x T_\infty}{\frac{kt}{\Delta x} + ht + \Delta x h}}$$

Can solve these eq.s to get T_i's using Gauss-Seidal

1) Guess values for T_i
2) Use node eqs to update T_i's.
3) Iterate until convergence

ex)



$T_0 = 100°C$ $\quad k = 170 \frac{W}{m\text{-}K}$

$\Delta x = 1.5cm$ $\quad h = 200 \frac{W}{m^2\text{-}K}$

$t = 0.002m$ $\quad T_\infty = 0$

Node eqs for interior nodes T_1, T_2, T_3:

$$T_i = \frac{\frac{kt}{\Delta x}(T_{i-1} + T_{i+1})}{\frac{2kt}{\Delta x} + 2h\Delta x} = \underline{\underline{0.442(T_{i-1} + T_{i+1})}}$$

Node eqn at tip:

$$T_n = \frac{\frac{kt}{\Delta x} T_{n-1}}{\frac{kt}{\Delta x} + ht + \Delta x h} \longrightarrow \underline{\underline{T_n = 0.869\, T_{n-1}}}$$

Gauss-Seidel: $\quad T_\infty = 0$

1) Assume $T_1 = T_2 = T_3 = T_4 = 0$

$$\left.\begin{aligned} &2)\ T_1 = 0.442(100 + \overset{T_2=0}{0}) = 44.2 \\ &T_2 = 0.442(T_1 + T_3) = 0.442(44.2 + 0) = 19.5 \\ &T_3 = 0.442(\overset{T_2=19.5}{T_2} + \underset{T_4=0}{T_4}) = 8.64 \\ &T_4 = 0.869\,T_3 = 7.50 \end{aligned}\right.$$

$$\underbrace{\qquad\qquad\qquad\qquad}_{1^{st}\ \text{Iteration}}$$

Excel:

T_0	T_1	T_2	T_3	T_4
100	0	0	0	0