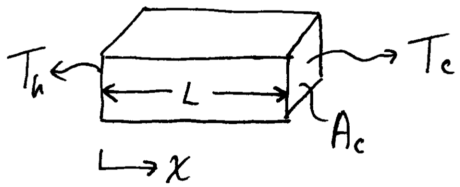
ENME332 - Heat Transfer

8/31/09

3 modes of heat transfer: conduction, convection, radiation

conduction - molecular vibrations / electronic transfer



$$\dot{q}_{[W]} \propto \frac{\Delta T}{L} A_c \qquad \Delta T = T_h - T_c$$

so we expect that $\boxed{\dot{q} = k A_c \frac{\Delta T}{L}}$ Fourier's Law

$\dot{q} = k A_c \frac{dT}{dx}$ as $L \to 0$

units: $[\dot{q}] = W = J/s$

$[A_c] = m^2$

$\left[\frac{dT}{dx}\right] = \frac{°C}{m} = \frac{K}{m}$

$[k] = \left[\frac{q}{A_c \frac{dT}{dx}}\right] = \frac{W}{m-K} = \frac{W}{m-°}$

nomenclature:

$q\,[W]$ $\quad q'\,[W/m]$

$\dot{q}\,[W/m^3]$ $\quad q''\,[W/m^2]$ $\quad$ (book)

overdot = "per unit time"

prime = "per unit length"

professor's		book's
$\dot{q}\,[W] \equiv [J/s]$	$\equiv$	q
$\dot{q}'''\,[W/m^3]$	$\equiv$	$\dot{q}$
$\dot{q}'\,[W/m]$	$\equiv$	q'
$\dot{q}''\,[W/m^2]$	$\equiv$	q''

property values in book:

thermal conductivity of air:

air properties

$T\,(K)$	$K \times 10^3\ \frac{W}{m-K}$
100	~
200	~
300	26.3

$K_{air} = 26.3\times10^{-3}\,\frac{W}{m}$

$K_{air} \neq 26.3\times10^{3}\,\frac{W}{m-K}$

gases:

- K_{gases} is independent of pressure

$K_{air} = 0.026\,\frac{W}{m-K}$ $\qquad$ $K_{helium} = 0.175\,\frac{W}{m-K}$

$K_{hydrogen} = 0.175\,\frac{W}{m-K}$ $\qquad$ $K_{CO_2} = 0.015\,\frac{W}{m-K}$

liquids:

- $K_{liquids}$ depends on temperature, but can increase/decrease depending on fluid & temperature

$$K_{water} = 0.556 \frac{W}{m\text{-}K}$$

$$oil = 0.147 \frac{W}{m\text{-}K}$$

$$mercury = 8.21 \frac{W}{m\text{-}K}$$

solids:

- lattice vibration electronic transport

K_{glass} : $1.5 \frac{W}{m\text{-}K}$

$K_{stainless\ steel}$: $15 \frac{W}{m\text{-}K}$

$K_{silicon}$: $120 \frac{W}{m\text{-}K}$

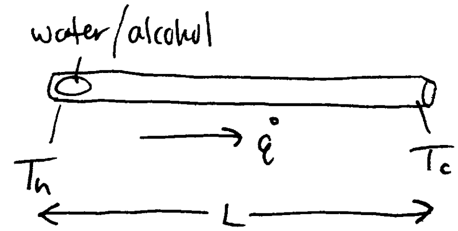
$K_{sapphire}$: $70 \frac{W}{m\text{-}K}$

$K_{aluminum}$: $200 \frac{W}{m\text{-}K}$

K_{copper} : $400 \frac{W}{m\text{-}K}$

$K_{diamond}$: $2000 \frac{W}{m\text{-}K}$

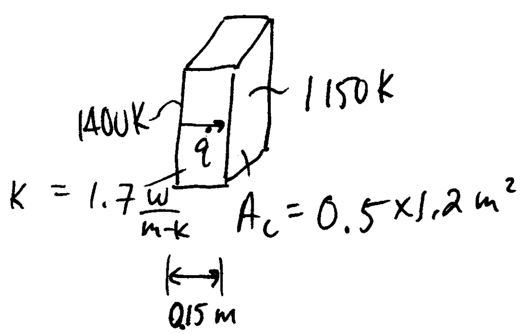
heat pipes: $K_{effective} \sim 20{,}000 \frac{W}{m\text{-}K}$



ex) walls of a furnace made of brick ($k = 1.7 \frac{W}{m\text{-}K}$) wall is 0.15 m thick

$T_{inner} = 1400$ K, $T_{outer} = 1150$ K

what is heat loss over a $0.5 \times 1.2\ m^2$ area?

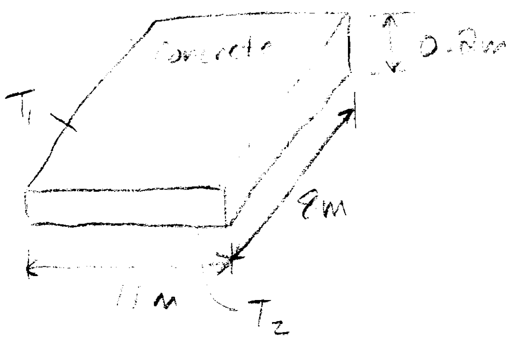


$$\dot{q} = kA \frac{dT}{dx}$$

$$= \left(1.7 \frac{W}{m\text{-}K}\right)(0.5m \times 1.2m)\left(\frac{-1400K + 1150K}{0.15m}\right)$$

$$= -1700\ W$$

1.3)



$k = 1.4 \frac{W}{m \cdot K}$

$T_1 = 17°C$

$T_2 = 10°C$

$\eta_f = 0.9$ efficiency

cost = \$0.01/MJ

$$\dot{q} = -k A_c \frac{\Delta T}{L}$$

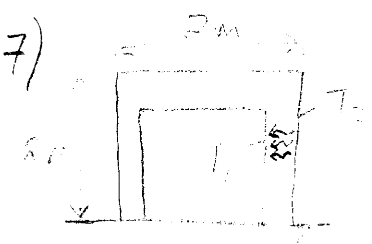
$$= -1.4 \frac{W}{m \cdot K} (11m \times 8m) \frac{(10-17)°C}{0.2m} = 4312 \text{ W}$$

$$\eta_f = \frac{\dot{q}_o}{\dot{q}_i} \rightarrow \dot{q}_i = \frac{\dot{q}_o}{\eta_f} \longrightarrow \dot{q}_i = \frac{4312 W}{0.9} = 4,791 \text{ W}$$

1 day = 86,400 s $\longrightarrow$ (4,791 J/s)(86400 s) = 414 MJ in 1 day

$$\$0.01/_{MJ} (414 MJ) = \boxed{\$4.14 \text{ per day}}$$

1.7)



$T_1 = -10°C$

$T_2 = 35°C$

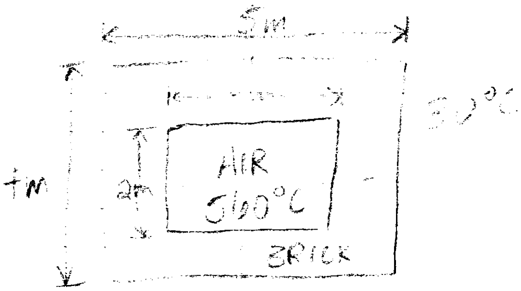
$k = 0.03 \frac{W}{m \cdot K}$

$\dot{q} = 500$ W

$$\dot{q} = -k A_c \frac{\Delta T}{L}$$

$$500 W = \left(-0.03 \frac{W}{m \cdot K}\right)(5 \times 2m \times 2m) \frac{(-10-35)°C}{t}$$

$$\boxed{t = 0.054 m}$$



$$\dot{q} = -hA_c\Delta T$$

heat transfer coefficient for convection

$h_{\text{ins}} = 10$

$h_{\text{outside}} = 5$

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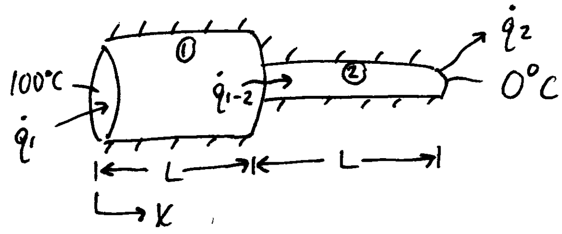
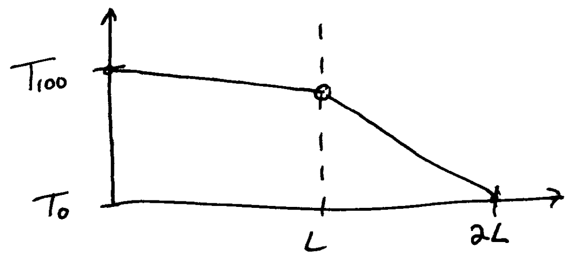
example for <u>Heat Conduction</u>:

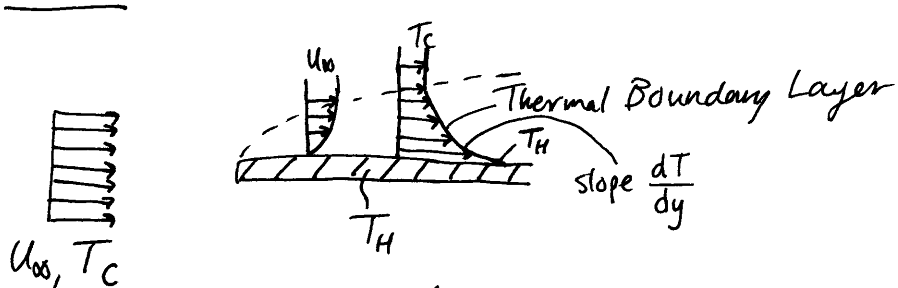
$$\dot{q} = -kA\frac{dT}{dx}$$





<u>Steady state</u>: all heat flowing must flow out

<u>Convection</u> - Heat transfer by bulk fluid motion

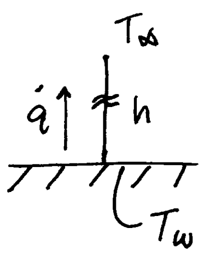


$$\dot{q} = -kA\frac{dT}{dy}$$

$$* \quad \dot{q} = hA_s(T_w - T_\infty)$$

A_s: surface area available for convection

h: heat transfer coefficient $[h] = \left[\frac{-\dot{q}}{A_s(T_w - T_\infty)}\right] = \frac{W}{m^2\text{-}K} \equiv \frac{W}{m^2\text{-}°C}$



$$\dot{q} = hA(T_w - T_\infty)$$

or $\dot{q}'' = \frac{\dot{q}}{A} = h(T - T_\infty)$

$\left(\frac{W}{m^2}\right)$ Heat Flux

<u>Types of Convection</u>:

Forced Convection - Fluid forced over a surface (wind tunnel) (Boiling & condensation

Free Convection - (Natural convection) ⇛

Free Convection in air : $h \cong 6-10 \ \frac{W}{m^2\text{-}K}$

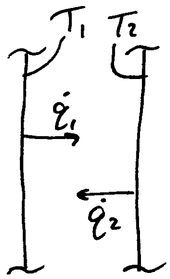
" " " water : $h \cong 1000 \ \frac{W}{m^2\text{-}K}$

Forced Convection in air : $h \cong 10-100 \ \frac{W}{m^2\text{-}K}$

" " " Water : $h \sim 3000 \ \frac{W}{m^2\text{-}K}$

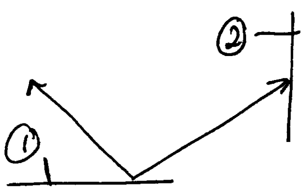
Pool Boiling in water : $2500 - 35{,}000 \ \frac{W}{m^2\text{-}K}$

Suppose we had two infinite plates facing each other :



The net radiation exchange between 1 & 2 is given by :

What if surfaces don't see each other completely?



use a view factor to account for this

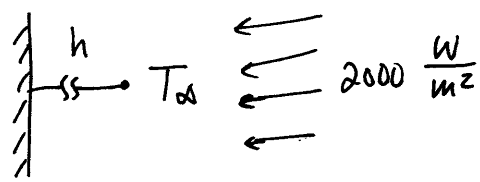
$$\dot{q}''_{1-2} = \sigma F_{ij} \left(T_1^4 - T_2^4\right)$$

view factor

Non-black body behavior is accounted for by an "emissivity"

$$\dot{q}''_{1-2} = \varepsilon \sigma F_{ij} \left(T_1^4 - T_2^4\right)$$

$0 < \varepsilon < 1$ $\qquad \varepsilon = 1 \Rightarrow$ black body ex) coating on a plate is cured by exposure to IR radiation that provides uniform illumination @ 2000 $\frac{W}{m^2}$. The surface is also exposed to an air flow @ 20°C. The surroundings are at 30°C. If the surface absorbs 80% of incoming radiation and has $\varepsilon = 0.50$, what is the temperature of the surface if $h = 15 \frac{W}{m^2\text{-}K}$?



* Assume S.S. $\rightarrow$ heat in = heat out

$\hookrightarrow$ Energy absorbed = Energy lost

$$2000 \frac{W}{m^2} \times 0.80 = h\left(T_w - T_\infty\right) + \varepsilon\sigma\left(T_w^4 - T_\infty^4\right)$$

$$2000 \frac{W}{m^2} \times 0.80 = 15 \frac{W}{m^2\text{-}K}\left(T_w - \underset{+273\,K}{20^\circ C}\right) + (0.50)\sigma\left(T_w^4 - \underset{+273K}{30^\circ C^4}\right)$$

<must be Kelvin> <must be Kelvin>

$$\boxed{T_w = 377\ K}$$

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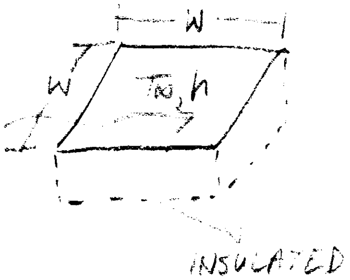
1.18) $w = 5\,mm$ $\quad h = 200\ \frac{W}{m^2\text{-}K}$ air coolant, maximum allowable power?

$T_\infty = 15°C$

$T_{max} = 85°C$



$$\dot{q} = -h A_c \Delta T$$

$$= \left(-200\ \frac{W}{m^2\text{-}K}\right)(0.005m \times 0.005m)^2 (15°C - 85°C)$$

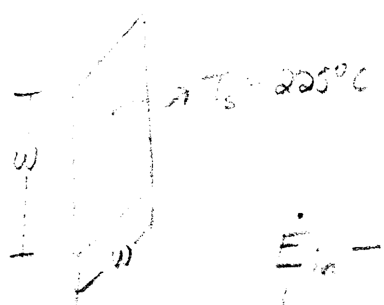
$$= \boxed{0.35\ W}$$

$h = 3000\ \frac{W}{m^2\text{-}K}$ dielectric liquid

$$\dot{q} = -\left(3000\ \frac{W}{m^2\text{-}K}\right)(0.005m \times 0.005m)^2 (15°C - 85°C)$$

$$= \boxed{5.25\ W}$$

1.22) Thin vertical plate suspended in still air



$\frac{dT}{dt} = -0.022\ K/s$

Ambient air $= 25°C$

$w = 0.3\ m$

$m = 3.75\ kg$

$c_p = 2770\ \frac{J}{kg\cdot K}$

$$\dot{E}_{in} - \dot{E}_{out} = \dot{E}_{change}$$

$$\underset{0}{\dot{E}_{in}} \quad \underset{h}{\dot{E}_{out}}$$

$$0 - hA(T_s - T_\infty) = m c_p \frac{dT}{dt} \longrightarrow \dot{E}_{in} = m c_p \frac{dT}{dt}$$

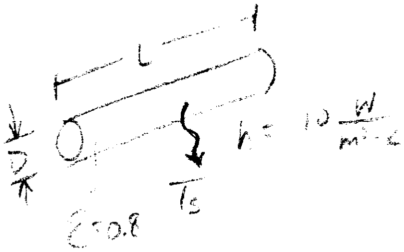
$$h = \frac{-m c_p \frac{dT}{dt}}{A(T_s - T_\infty)} = \frac{(-3.75\,kg)(2770\ \frac{J}{kg\text{-}K})(-0.022\ K/s)}{(0.3\,m \times 0.3\,m)(225°C - 25°C)}$$

$$= 12.7$$

1.22) Uninsulated steam pipe   surrounding walls & air at 25°C

$L = 25\,m$ pipe surface temperature of 150°C

$D = 100\,mm$ $h = 10\,\frac{W}{m^2\text{-}K}$, $\varepsilon = 0.8$



a) rate of heat loss?

$\dot{q}_{conv} = -hA\Delta T$

$= (-10\,\frac{W}{m^2 K})(\pi)(0.1\,m)(25\,m)(\ldots - \ldots 5°C)$

$= 9.87\ W$ view factor

$\dot{q}_{rad} = \varepsilon\sigma(T_s^4 - T_\infty^4)F$

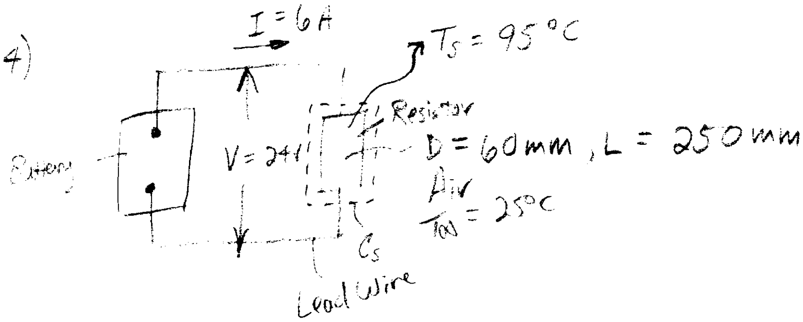
$= 0.8(\ldots)(423\,K^4 - 298\,K^4)(\pi)(0.1\,m)(25\,m)$

$=$

b) $\eta_f = 0.90$, $C_g = \$0.01/MJ$ annual cost of heat loss?

$E = 5.8 \times 10^{11}\,J = 18450 \times 3600 \times 24 \times 365$

1.34)



a) Find $\dot{E}_{in}$, $\dot{E}_g$, $\dot{E}_{out}$, $\dot{E}_{st}$

$$\dot{E}_{st} \equiv \frac{dE_{st}}{dt} = \dot{E}_{in} - \dot{E}_{out} + \dot{E}_g$$

$\dot{E}_{in} = 0$
$\dot{E}_{out} = 144\,W$
$\dot{E}_{st} = 0$
$\dot{E}_g = IV = 144\,W$

$\}$ control surface of Resistor

b) Volumetric heat generation rate?

=

=

$= 2.04 \times 10^5 \frac{W}{m^3}$

c) h?

$$\dot{E}_{out} = hA_{tot}(T_s - T_\infty)$$

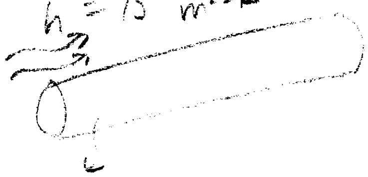
$$144\,W = h\left(\pi DL + \left(\frac{\pi}{4}D^2\right)\times 2\right)(95°C - 25°C)$$

$$h = 38.98 \frac{W}{m^2 \cdot K}$$

1.64)

$T_\infty = 25°C$

$h = 15 \frac{W}{m^2 \cdot K}$



$T_s = 200°C$

$D = 70mm$

$\varepsilon = 0.8$

a) $G = \sigma T_{sur}^4$

$$\frac{E_{net}}{L} = E_{rad} - \alpha G$$

$$= \pi D \left[\varepsilon \sigma (T_s^4) - \alpha \sigma T_{sur}^4 \right]$$

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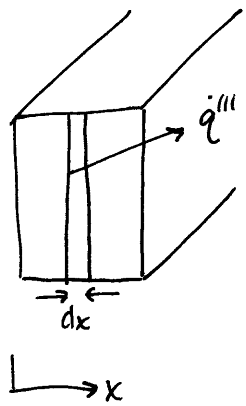
Heat of Diffusion Eq.



$$\dot{q}_{in} + \dot{q}_{gen} - \dot{q}_{out} = \dot{q}_{st}$$

$$\dot{q}_{in} = \dot{q}_x = -kA\frac{\partial T}{\partial x}$$

$$\dot{q}_{gen} = \dot{q}'''A\,dx \qquad \dot{q}''' = \left[\frac{W}{m^3}\right]$$

$$\dot{q}_{out} = -kA\frac{\partial T}{\partial x}\bigg|_{x+dx}$$

$$= -kA\left[\frac{\partial T}{\partial x} + \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)dx\right]$$

$$\dot{q}_{st} = mc\frac{\partial T}{\partial t}$$

$$\cancel{-kA\frac{\partial T}{\partial x}} + \dot{q}'''A\,dx - \cancel{kA\frac{\partial T}{\partial x}} + A\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) = \dot{q}_{st} = \rho c_p\frac{\partial T}{\partial t}A\,dx$$

$$\dot{q}'''A\,dx + A\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) = \rho c_p\frac{\partial T}{\partial t}A\,dx$$

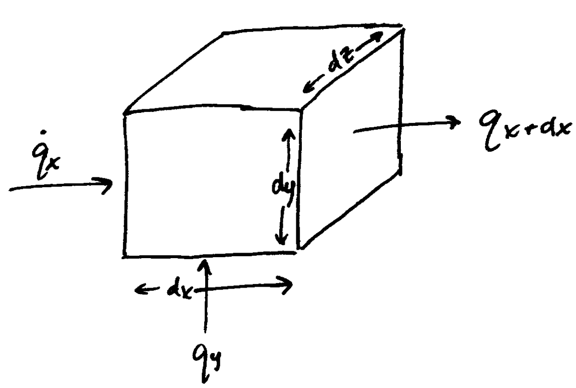
1-D

$$\boxed{\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \dot{q}''' = \rho c_p\frac{\partial T}{\partial t}}$$

- Heat of Diffusion Eq.

3-D

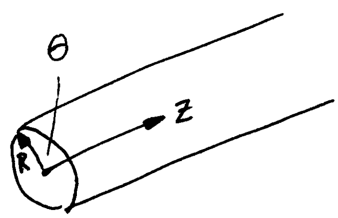
$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{q}''' = \rho c_p\frac{\partial}{\partial}$$



$k = \text{constant}$ ⟨divide thru by k⟩

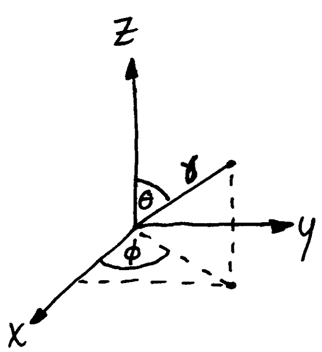
$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}'''}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau} \qquad \alpha = \frac{k}{\rho c_p} = \text{thermal diff}$$

Cylindrical Coordinates



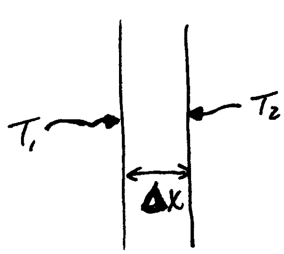
$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}'''}{k} = \frac{1}{\gamma}\frac{\partial T}{\partial \tau}$$

Spherical Coordinates



$$\frac{1}{r}\frac{d^2}{dr^2}(rT) + \frac{1}{r^2 \sin\theta}\frac{d}{d\theta}\left(\sin\theta \frac{dT}{d\theta}\right) + \frac{1}{r^2 \sin^2\theta}\left(\frac{d^2 T}{d\phi^2}\right)$$
$$+ \frac{\dot{q}'''}{k} = \frac{1}{r}\frac{dT}{d\tau}$$

Steady State 1-D Convection



$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) = 0$$

$$k\frac{\partial T}{\partial x} = D$$

$$\frac{\partial T}{\partial x} = A \leftarrow D/k = A$$

integrate → $T = Ax + B$

Boundary Conditions:

$T = T_1$ @ $x = 0$

$T = T_2$ @ $x = \Delta x$

$T_1 = A \times 0 + B = B$

$T_2 = A\Delta x + B$

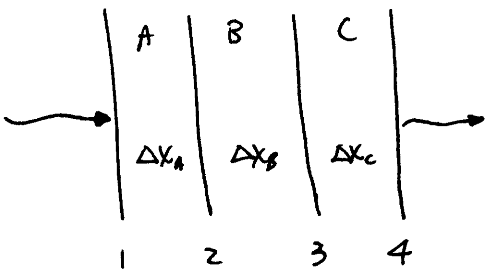
$$T = \frac{(T_2 - T_1)x}{\Delta x} + T_1$$

$$k = k_0(1 + BT)$$

- not a fcn. of x, y, z
- a function of T

$$q = -\frac{k_0 A}{\Delta x}\left[(T_2 - T_1) + \frac{B}{2}(T_2^2 - T_1^2)\right]$$

Composite Wall (s.s.)



$$q = -K_A A_A \frac{(T_2 - T_1)}{\Delta X_A} \qquad A_A = A_B = A_C$$

$$= -K_B A_B \frac{(T_3 - T_2)}{\Delta X_B}$$

$$= -K_C A_C \frac{(T_4 - T_3)}{\Delta X_C}$$

$$(T_1 - T_2) + (T_2 - T_3) + (T_3 - T_4) = \frac{q}{A}\left[\frac{\Delta X_A}{K_A} + \frac{\Delta X_B}{K_B} + \frac{\Delta X_C}{K_C}\right]$$

$$q = -\frac{K_{eff} A (T_4 - T_1)}{\Delta X_A + \Delta X_B + \Delta X_C}$$

$$K_{eff} = \frac{\Delta X_A + \Delta X_B + \Delta X_C}{\frac{\Delta X_A}{K_A} + \frac{\Delta X_B}{K_B} + \frac{\Delta X_C}{K_C}}$$

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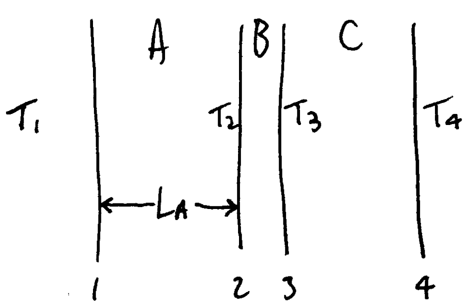


$$\dot{q}_A = \dot{q}_B = \dot{q}_C$$

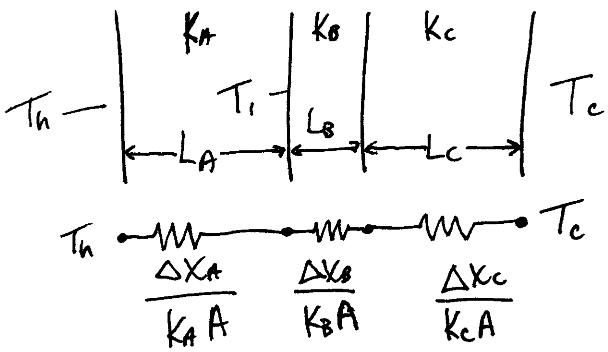
$$-K_A \frac{dT}{dx}\bigg|_A = -K_B \frac{dT}{dx}\bigg|_B = -K_C \frac{dT}{dx}\bigg|_C$$

Electrical Resistance Analogy

Electricity	Heat Transfer
I	$\dot{q}$
ΔV	ΔT
R	R_{th}

$$\dot{q} = -kA\frac{\Delta T}{\Delta x} \equiv I = \frac{V}{R}$$

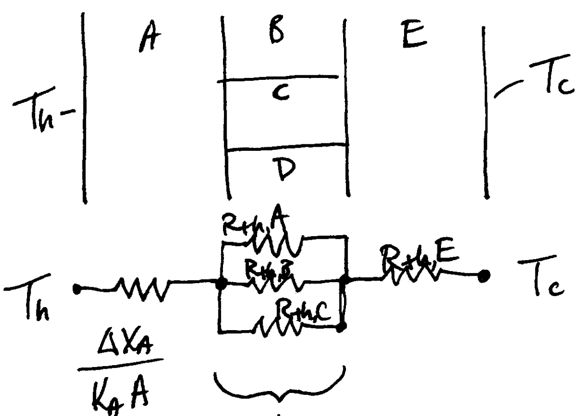
$$= \frac{\Delta T}{R_{th}} \longrightarrow \boxed{R_{th} = \frac{\Delta x}{kA}}$$



$$\dot{q} = \frac{T_h - T_c}{\sum R_{th}}$$

$$T_1: \quad \dot{q} = \frac{T_h - T_1}{\frac{\Delta X_A}{k_A A}}$$

$\dot{q} \longrightarrow$ 1-D

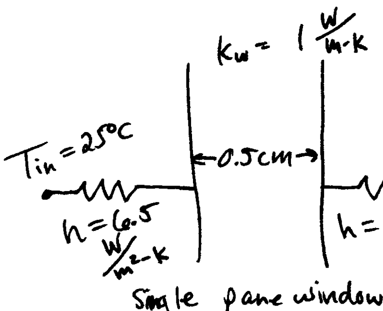


$$\frac{1}{R_{eq}} = \frac{1}{R_{th,A}} + \frac{1}{R_{th,B}} + \frac{1}{R_{th,C}}$$

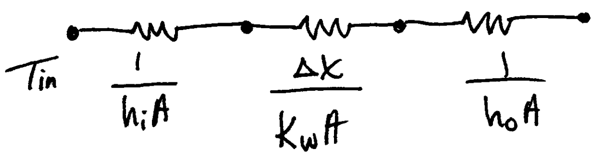
Convection Resistances

$$\dot{q} = hA(T_w - T_\infty)$$

$$= \frac{T_w - T_\infty}{\frac{1}{hA}} = \frac{T_w - T_\infty}{R_{th}} \longrightarrow R_{th} = \frac{1}{hA}$$



what is the heat flow per unit area?

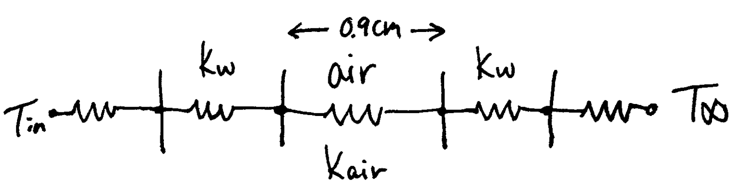


$$\dot{q} = \frac{T_i - T_\infty}{\frac{1}{h_i A} + \frac{\Delta x}{k_w A} + \frac{1}{h_o A}} \Rightarrow \frac{\dot{q}}{A} = \dot{q}'' = \frac{T_i - T_\infty}{\frac{1}{h_i} + \frac{\Delta x}{k_w} + \frac{1}{h_o}}$$

$$= \frac{25°C - 0°C}{\frac{1}{6.5 \frac{W}{m^2\text{-}K}} + \frac{0.005 m}{1 \frac{W}{m\text{-}K}} + \frac{1}{20 \frac{W}{m^2\text{-}K}}} = 120 \frac{W}{m^2}$$

---

double pane window:

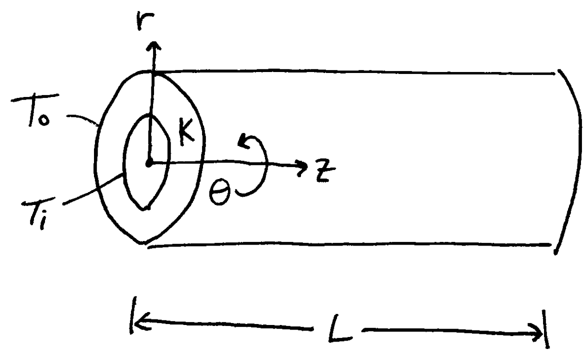


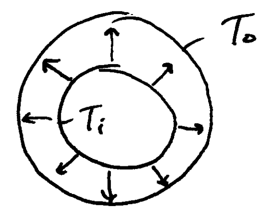
$$\dot{q}'' = \frac{\dot{q}}{A} = \frac{T_i - T_\infty}{\frac{1}{h_i} + \frac{\Delta x_w}{k_w} + \frac{\Delta x_{air}}{k_{air}} + \frac{\Delta x_w}{k_w} + \frac{1}{h_o}}$$

$$\dot{q}'' = 44 \frac{W}{m^2}$$

Cylindical Systems

- consider a long hollow cylinder of length L, inner radius r_i, outer radius r_o





$$\dot{q} = -kA\frac{dT}{dr}$$

$$A(r) = 2\pi r L$$

$$\dot{q} = \frac{-k2\pi \cancel{r} L}{\cancel{r}}\ \frac{T_o - T_i}{\ln(r_o/r_i)}$$

gov. eq: $\dfrac{d^2T}{dr^2} + \dfrac{1}{r}\dfrac{dT}{dr} + \dfrac{1}{r^2}\dfrac{d^2T}{d\theta^2} + \dfrac{d^2T}{dz^2} + \dfrac{\dot{q}'''}{k} = \dfrac{1}{\alpha}\dfrac{dT}{dt}$

only dependent on radius: $\dfrac{d^2T}{dr^2} + \dfrac{1}{r}\dfrac{dT}{dr} = 0$

$$\frac{d}{dr}\left(\frac{dT}{dr}\right) + \frac{1}{r}\frac{dT}{dr} = 0$$

$$\int \frac{d\left(\frac{dT}{dr}\right)}{\frac{dT}{dr}} = \int -\frac{1}{r}\,dr$$

$$\rightarrow \ln\left(\frac{dT}{dr}\right) = -\ln(r) + A \rightarrow \frac{dT}{dr} = C\frac{1}{r}$$

$$T = C\ln(r) + D$$

use B.C.: $T = T_i$ @ $r = r_i$, $T = T_o$ @ $r = r_o$

$$\boxed{\frac{T - T_i}{T_o - T_i} = \frac{\ln(r/r_i)}{\ln(r_o/r_i)}}$$

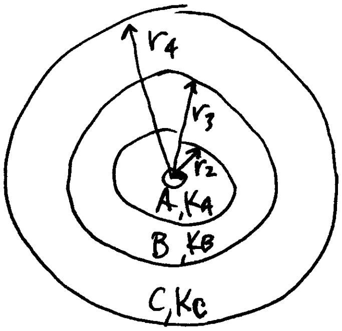
Temp. distribution in cylinder

$$\dot{q} = \frac{T_i - T_o}{\frac{\ln(r_o/r_i)}{2\pi kL}} = \frac{T_i - T_o}{R_{th}}$$

$$\boxed{R_{th} = \frac{\ln(r_o/r_i)}{2\pi kL}}$$

9/16/09

Multi-Layered Cylinders



r_3, T_3

r_4, T_4

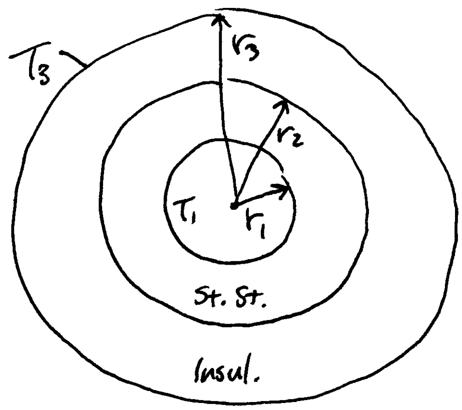
r_2, T_2

r_1, T_1

$$\boxed{R_{th} = \frac{\ln\left(\frac{r_o}{r_i}\right)}{2\pi K L}}$$

$$\dot{q} = \frac{\Delta T_{overall}}{R_{th,A} + R_{th,B} + R_{th,C}}$$

$$= \frac{(T_1 - T_4)\,2\pi L}{\frac{\ln\left(\frac{r_2}{r_1}\right)}{K_A} + \frac{\ln\left(\frac{r_3}{r_2}\right)}{K_B} + \frac{\ln\left(\frac{r_4}{r_3}\right)}{K_C}}$$



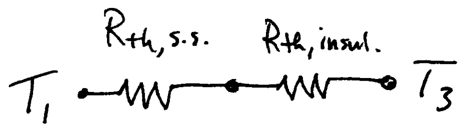
$T_1 = 600°C$

$T_3 = 100°C$

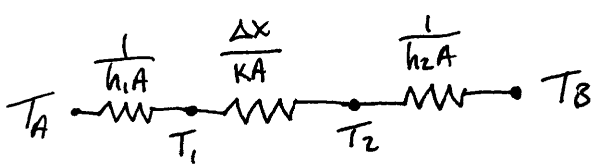
$r_1 = 0.01\,m$

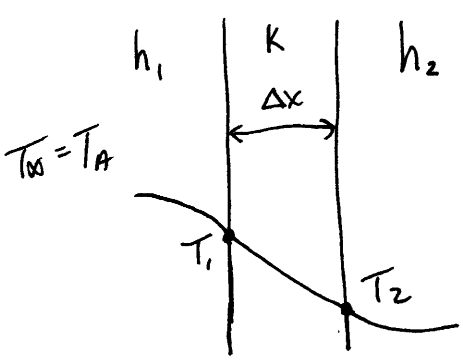
$r_2 = 0.02\,m$

$r_3 = 0.05\,m$



Overall H.T. Coeff.





$T_\infty = T_B$

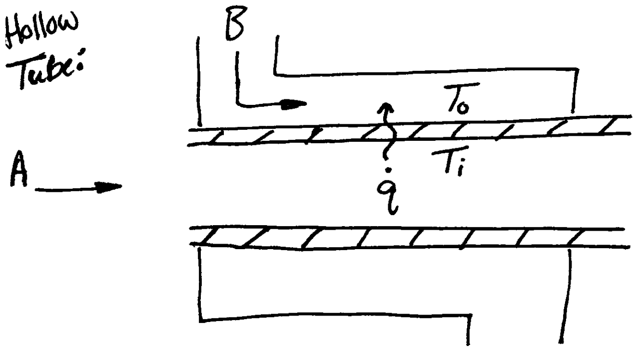
$$\dot{q} = \frac{\Delta T}{\Sigma R_{th}} = \frac{T_A - T_B}{\frac{1}{h_1 A} + \frac{\Delta x}{K A} + \frac{1}{h_2 A}}$$

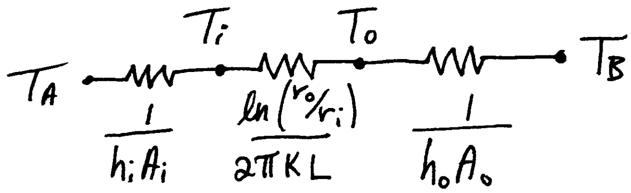
$\longrightarrow \dot{q} = U A \Delta T_{overall}$

U - overall H.T. coeff.

$$\boxed{U = \frac{1}{\frac{1}{h_1} + \frac{\Delta x}{K} + \frac{1}{h_2}}}$$

Parallel Flow Heat Exchanger





$$A_i = 2\pi r_i L \qquad A_o = 2\pi r_o L$$

$$\dot{q} = \frac{\Delta T_{overall}}{\Sigma R_{th}} = \frac{T_A - T_B}{\frac{1}{h_i A_i} + \frac{\ln(r_o/r_i)}{2\pi K L} + \frac{1}{h_o A_o}}$$

$$U_i = \frac{\dot{q}}{A_i (T_A - T_B)}$$

$$U_i = \frac{1}{\frac{1}{h_i} + \frac{A_i \ln(r_o/r_i)}{2\pi K L} + \frac{A_i}{h_o A_o}}$$

$$U_o = \frac{\dot{q}}{A_o (T_A - T_B)}$$

$$U_o = \frac{1}{\frac{A_o}{A_i h_i} + \frac{A_o \ln(r_o/r_i)}{2\pi K L} + \frac{1}{h_o}}$$

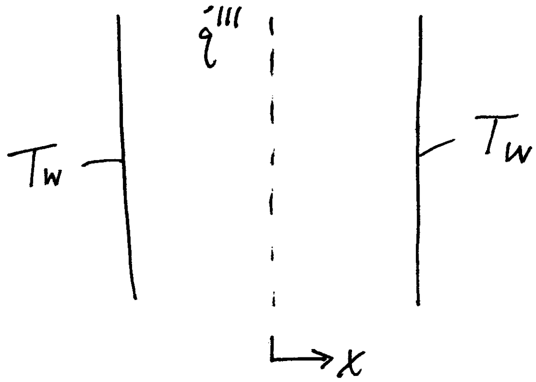
R Value for Insulation

$$R = \frac{\Delta T}{\dot{q}/A} = \frac{\Delta T}{\frac{KA\Delta T}{\Delta x A}} = \frac{\Delta x}{K} \quad \begin{matrix}\uparrow \\ \downarrow\end{matrix} \qquad R\uparrow$$

- 6" Fiberglass $\quad R = 3 \frac{W}{m^2 K}$

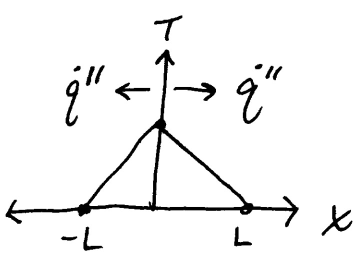
9/21/09

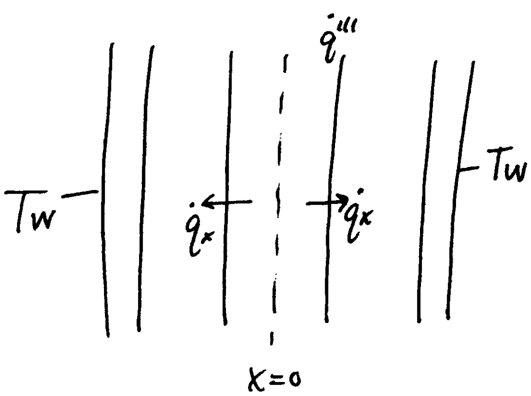
Internal Heat Generation (Cannot use resistance network analogy)

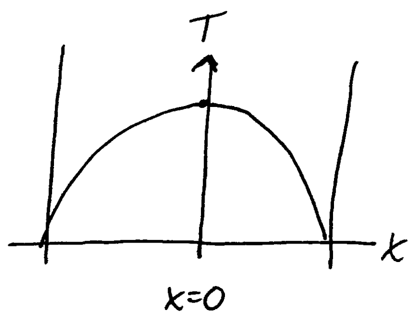


steady state

$\dot{q}''' = \text{const.}$







Derive Temperature distribution in slab:

$$\frac{d}{dx}\left(k\frac{\partial T}{\partial x}\right) + \cancelto{0}{\frac{d}{dy}\left(k\frac{\partial T}{\partial y}\right)} + \cancelto{0}{\frac{d}{dz}\left(k\frac{\partial T}{\partial z}\right)} + \dot{q}''' = \cancelto{0}{\rho c_p \frac{dT}{dt}}$$

s.s.

$$\frac{d}{dx}\left(k\frac{\partial T}{\partial x}\right) = -\dot{q}''' \qquad k = \text{const.}$$

$$\frac{d}{dx}\left(\frac{\partial T}{\partial x}\right) = -\frac{\dot{q}'''}{k} \longrightarrow d\left(\frac{dT}{dx}\right) = -\frac{\dot{q}'''}{k}dx$$

$$\frac{dT}{dx} = -\frac{\dot{q}'''}{k}x + A$$

$$T = -\dot{q}'''\frac{x^2}{2k} + Ax + B$$

B.C. : $T = T_w$ @ $x = L$, $T = T_w$ @ $x = -L$

Solve for A, B to obtain

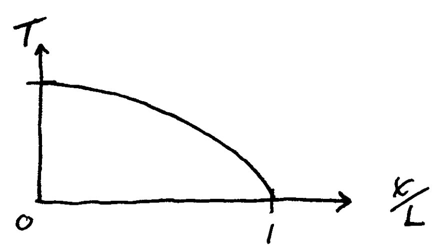
$$T - T_w = -\frac{\dot{q}'''}{2k}(x^2 - L^2)$$

T_{max} occurs @ $x = 0$: $T_{max} = T_w + \frac{\dot{q}''' L^2}{2k}$

or $T_{max} - T_w = +\frac{\dot{q}''' L^2}{2k}$

Non-dimensional temperature profile:

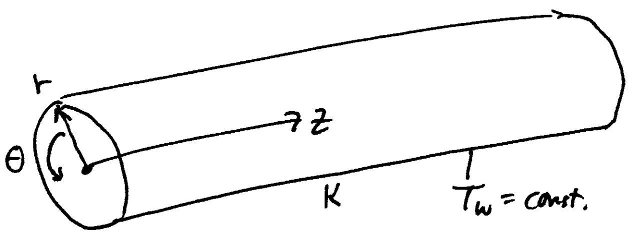
$$\frac{T - T_w}{T_{max} - T_w} = 1 - \frac{x^2}{L^2}$$



Cylindrical Systems

– solid cylinder, radius R, $\dot{q}''' = \text{const.}$, S.S.

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}'''}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$



symmetric axis, S.S., $\dot{q}''' = \text{const.}$

$$\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} = -\frac{\dot{q}'''}{K}$$

B.C.: $T = T_w$ @ $r = R$, $\frac{\partial T}{\partial r} = 0$ @ $r = 0$

Alternate B.C.: All energy generated in cylinder must be transferred by conduction at the wall.

$$\dot{q}''' \times V = \dot{q}''' \times \pi R^2 L = K A_s \left.\frac{dT}{dr}\right|_{r=R}$$

REWRITE

$$\dot{q}''' \pi R^2 L = K\, 2\pi R L \left.\frac{dT}{dr}\right|_{r=R}$$

$$\dot{q}''' R = 2K \left.\frac{dT}{dr}\right|_{r=R} \longleftarrow \text{Alt. B.C.}$$

~~then~~ $\cancel{\frac{d}{dr}\left(r\frac{dT}{dr}\right) = \frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr}}$

$$\underbrace{r\frac{d^2T}{dr^2} + \frac{dT}{dr}}_{} = -\frac{\dot{q}''' r}{K}$$

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = \frac{-\dot{q}''' r}{K} \xrightarrow{\text{INTEGRATE}} r\frac{dT}{dr} = -\frac{\dot{q}''' r^2}{2K} + C$$

Apply B.C.: $\frac{dT}{dr} = 0$ @ $r = 0 \rightarrow C = 0$

$$\frac{dT}{dr} = \frac{-\dot{q}'''r}{2K} \longrightarrow T = \frac{-\dot{q}'''r^2}{4K} + C_2$$

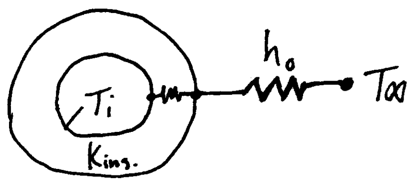
B.C.: $T = T_w$ @ $r = R \longrightarrow C_2 = T_w + \frac{\dot{q}'''R^2}{4K}$

$$T - T_w = \frac{\dot{q}'''}{4K}\left(R^2 - r^2\right)$$

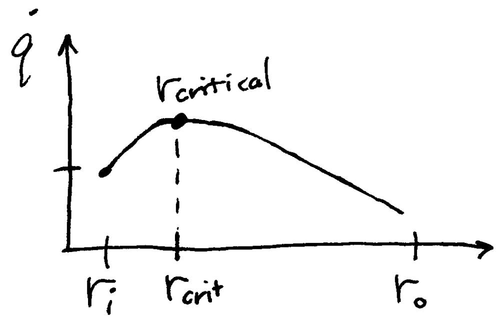
@ $r = 0$ $\quad T_0 = \frac{\dot{q}'''R^2}{4K} + T_w$ (max Temp. in cylinder)

discussion 9/22/09

Critical Radius of Insulation



$$\longrightarrow \quad T_i \; \underset{\frac{\ln(r_o/r_i)}{2\pi KL}}{\text{---}} \; \underset{\frac{1}{2\pi r_o hL}}{\text{---}} \; T_\infty$$



$$\dot{q} = \frac{T_i - T_\infty}{\frac{\ln(r_o/r_i)}{2\pi KL} + \frac{1}{2\pi L h r_o}}$$

To find r_{crit}: $\frac{d\dot{q}}{dr_o} = 0$ $\quad \boxed{r_{crit} = \frac{K}{h}}$

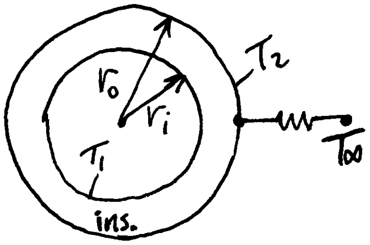
$$= \frac{(T_i - T_\infty)2\pi LKh}{h\ln(r_o/r_i) + K/r_o} = \frac{(T_i - T_\infty)2\pi LKh}{h(\ln r_o - \ln r_i) + K/r_o} = \frac{(T_i - T_\infty)2\pi LKh\, r_o}{h\ln(r_i) r_o}$$

$\ln r_o - \ln r_i$

$$\frac{d\dot{q}}{dr_o} = \frac{d}{dr_o}\left((T_i - T_\infty)\left(\frac{\ln(r_o/r_i)}{2\pi KL} + \frac{1}{2\pi L h r_o}\right)^{-1}\right)$$

$$= (T_\infty - T_i)\Big($$

ex)



$T_1 = 200°C$ $\quad h = 3 \frac{W}{m^2 \cdot K}$ $\quad r_i = 2.5\ cm$

$T_\infty = 20°C$ $\quad K_{ins} = 0.17 \frac{W}{m \cdot K}$

– what is $\dot{q}$ w/o insulation?

$\cancel{\dot{q} = K_{ins} A_s}$ $\quad \dot{q} = h A_s (T_1 - T_\infty)$

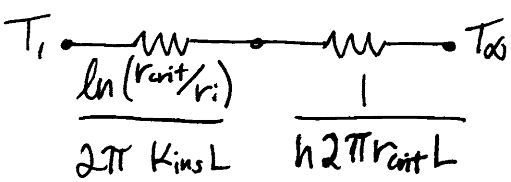
$$\frac{\dot{q}}{L} = 3 \frac{W}{m^2 K} \times 2\pi(0.025m)(200°C - 20°C)$$

$$= 84.8\ W/m$$

– what is $\dot{q}$ w/ r_{crit} of insulation

$$r_{crit} = \frac{k_{ins}}{h} = \frac{0.17 \frac{W}{m \cdot K}}{3 \frac{W}{m^2 \cdot K}} = 0.0567\ m$$

$$\cancel{\frac{\dot{q}}{L} = K_{ins} A_s \frac{(T_1 - T_2)}{r_{crit} - r_i} = 0.17 \frac{W}{m \cdot K} \times 2\pi(\quad) \frac{(200°C - T_2)}{(0.0567 - 0.025)m}}$$

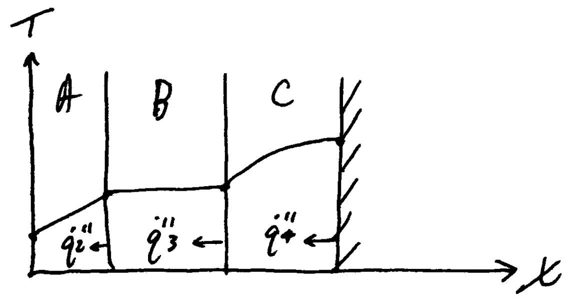


$$\frac{\dot{q}}{L} = \frac{T_1 - T_\infty}{\sum R_{th}} = 106\ W/m$$

Internal Heat Generation

$$\dot{q} = KA\frac{\Delta T}{\Delta x} \rightarrow \Delta T(\Delta x) = \frac{\dot{q}}{KA}\Delta x$$

S.S. heat generation in a plane wall of three different materials, each w/ constant K.



a) comment on the relative magnitudes of $\dot{q}_2''$, $\dot{q}_3''$, & $\dot{q}_4''$

b) comment on relative magnitudes of $K_A, K_B,$ & K_C

c) sketch $\dot{q}''(x)$

$$T(x) = \frac{\dot{q}}{K}\cancel{A}\, x \qquad \dot{q}'' = K\frac{\Delta T}{\Delta x} \rightarrow T(x) = \frac{\dot{q}''}{K}\, x$$

a) $\dot{q}_4'' = 0$

Inside C: $\dot{q}''' \neq 0 \rightarrow$ internal heat generation

The curve in C indicates internal heat generation

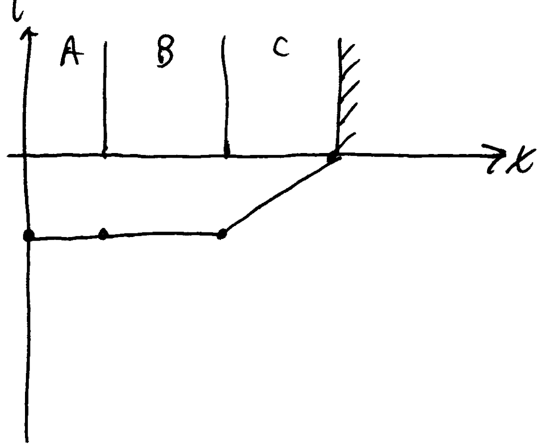
$\dot{q}_2'' = \dot{q}_3''$

b) $K_A < K_B$ $\qquad K_B > K_C$

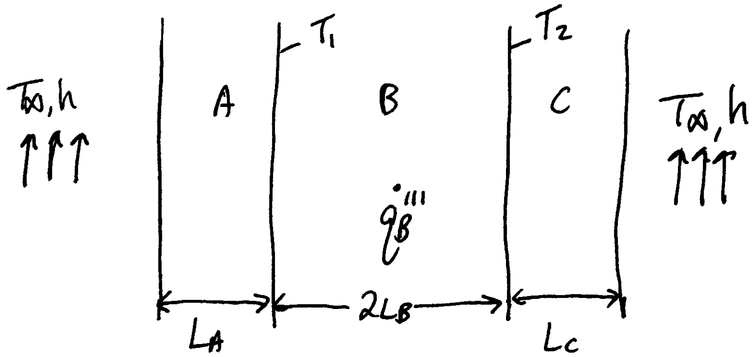
$\dot{q}'' = \dot{q}_2'' = \dot{q}_3''$

$K_A \frac{dT}{dx_A} = K_B \frac{dT}{dx_B}$

c) $\dot{q}_x''$



3.73)



$T_\infty = 25°C$

$h = 1000 \frac{W}{m^2\text{-}K}$

internal heat generat.

$\dot{q}_B'''$

$T_1 = 261°C$, $T_2 = 211°C$

$K_A = 25 \frac{W}{m\text{-}K}$ $\quad L_A = 30mm$

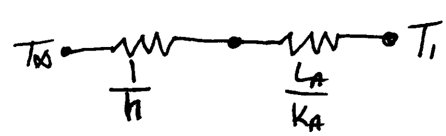
$K_c = 50 \frac{W}{m\text{-}K}$ $\quad L_B = 30\,mm$

$L_c = 20mm$

a) assuming negligible contact resistance at the interfaces, determine $\dot{q}_B'''$ & K_B

$$\dot{q}''' = K \frac{1}{L}\frac{dT}{dx} = \frac{\dot{q}''}{L} \longrightarrow \dot{q}'' = \dot{q}''' \times L \longrightarrow \boxed{\dot{q}_B''' L_B = \dot{q}_1'' + \dot{q}_2''}$$

$$\cancel{\dot{q}_A'' = K_A \frac{T_1 - T_\infty}{L_A} = 25 \frac{(261-25)}{0.03} = 197 \frac{KW}{m^2}}$$



$$\dot{q}_1'' = \frac{T_1 - T_\infty}{\Sigma R_{th}} = \frac{534K - 298K}{\frac{1}{1000 \frac{W}{m^2\text{-}K}} + \frac{0.03\,m}{25 \frac{W}{m\text{-}K}}} = 107 \frac{KW}{m^2}$$

$$\dot{q}_2'' = \frac{484K - 298K}{\frac{0.02m}{50 \frac{W}{m\text{-}K}} + \frac{1}{1000 \frac{W}{m^2\text{-}K}}} = 133 \frac{KW}{m^2}$$

$$\dot{q}_B''' = \frac{\dot{q}_1'' + \dot{q}_2''}{2L_B} = \frac{107{,}000 + 133{,}000}{2(0.03)} = \boxed{4 \frac{MW}{m^3}}$$

$$\cancel{2L_B \dot{q}_B''' = K_B \frac{dT}{dx} \rightarrow K_B =}$$

$$T(x) = \frac{-\dot{q}''' x^2}{2K_B} + C_1 x + C_2$$

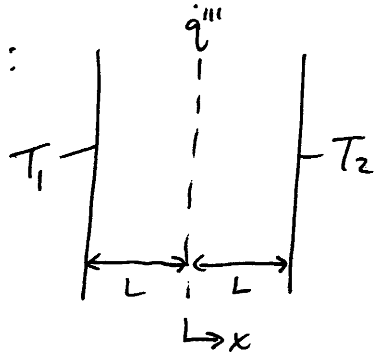
$$\left[\frac{\partial T(x)}{\partial x}\right] = \left[\frac{-\dot{q}''' x}{K_B} + C_1\right] K_B = \dot{q}''' x - C_1 K_B$$

9/23/09

Recap: Int. Heat Generation

S.S., $\dot{q}''' = \text{const.}$, $k = \text{const.}$

Planar Systems:



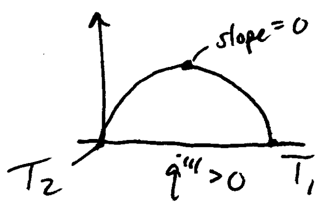
$$\boxed{\begin{array}{l} \dfrac{d^2T}{dx^2} + \dfrac{\dot{q}'''}{k} = 0 \\ T = \dfrac{-\dot{q}''' x^2}{2k} + C_1 x + C_2 \\ C_1, C_2 \text{ obtained from B.C.} \end{array}}$$

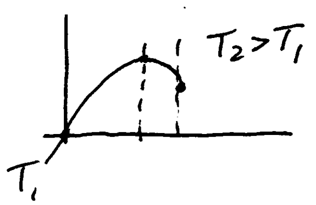
$$T(x) = \frac{\dot{q}''' L^2}{2k}\left(1 - \frac{x^2}{L^2}\right) + \frac{T_2 - T_1}{2}\,\frac{x}{L} + \frac{T_1 + T_2}{2}$$

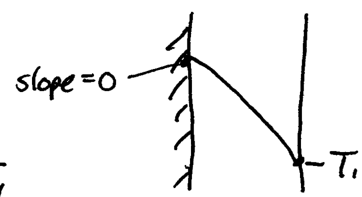
$$T_1 = T_2 = T$$

$$T(x) = \frac{\dot{q}''' L^2}{2k}\left(1 - \frac{x^2}{L^2}\right) + T$$

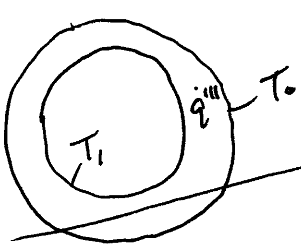
T Profiles







~~Radial Systems~~



$$T = T_o + \frac{\dot{q}'''}{4k}\left(r_o^2 - r_i^2\right) + C\frac{r}{r_o}, \quad C = \frac{T_i - T_o + \dot{q}'''\left(r_i^2 - r_o^2\right)/4k}{\ln\left(r_o/r_i\right)}$$

Cylinders w/ Heat Generation

- Uniformly distributed heat sources
- constant conductivity

$$\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} + \frac{\dot{q}'''}{k} = 0 \qquad \text{— gov. eq.}$$

B.C.: $T = T_w$ @ $r = R$, $\frac{dT}{dr} = 0$ @ $r = 0$

Alt. B.C.: Heat flux at surface = energy generated within the solid

$$\dot{q}'''\pi R^2 L = -k 2\pi R L \left.\frac{dT}{dr}\right|_{r=R}$$

$$r\frac{d^2T}{dr^2} + \frac{dT}{dr} = -\frac{\dot{q}'''r}{k}$$

-or- $$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = \frac{-\dot{q}'''r}{k} \xrightarrow{\text{integrate}} r\frac{dT}{dr} = \frac{-\dot{q}'''r^2}{2k} + A$$

B.C.: $0 = \frac{dT}{dr}$ @ $r = 0$ $\rightarrow A = 0$

$$\xrightarrow{\text{integrate}} T = \frac{-\dot{q}'''r^2}{4k} + B$$

B.C.: At $r = R$, $T = T_w$

$$\boxed{T - T_w = \frac{\dot{q}'''}{4k}(R^2 - r^2)}$$

Non-dimensional:

$$\boxed{\frac{T - T_w}{T_0 - T_w} = 1 - \left(\frac{r}{R}\right)^2}$$

$T_0 = T(r=0)$

$$T_0 = \frac{\dot{q}'''R^2}{4k} + T_w$$

Hollow Cylinder

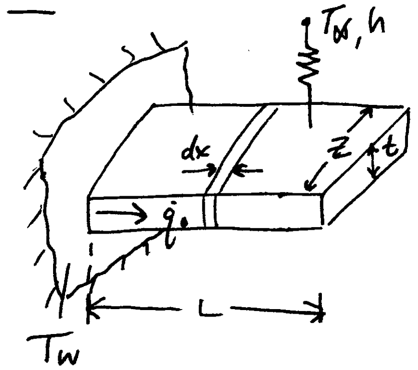
B.C.: $T = T_i$ @ $r = r_i$, $T = T_0$ @ $r = r_0$

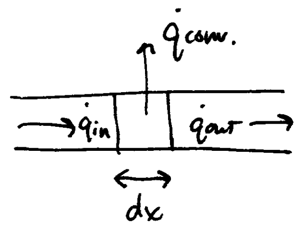
$$\boxed{T - T_0 = \frac{\dot{q}'''}{4k}(r_0^2 - r^2) + A \ln\left(\frac{r}{r_0}\right)}$$

$$\boxed{A = \frac{T_i - T_0 + \dot{q}'''(r_i^2 - r_0^2)/4k}{\ln(r_i/r_0)}}$$

9/28/09

Fins





Energy Balance: $\dot q_{in} = \dot q_{out} + \dot q_{conv}$

$$\dot q_{in}(x) = -kA\frac{dT}{dx} = -k(zt)\frac{dT}{dx}$$

$$\dot q_{out}(x) = \dot q_{in} + \frac{d}{dx}\left(\dot q(x)\right)dx \qquad \text{– Taylor Expansion}$$

$$\dot q_{conv} = hA_s\Delta T = h\underbrace{(2z+2t)}_{\text{perim.}}dx\,(T(x) - T_\infty)$$

E. Balance: $\cancel{\dot q_{in}(x)} = \cancel{\dot q_{in}} + \frac{d}{dx}(\dot q(x))\,dx + hP\,dx\,(T(x) - T_\infty)$

solve to get: $\boxed{\dfrac{d^2T}{dx^2} - \dfrac{hP}{kA}(T - T_\infty) = 0}$

Some B.C.

1) Fin is infinitely long: T at tip $= T_\infty$
2) Fin tip is insulated: $\frac{dT}{dx} = 0$ @ $x = L$
3) Fin loses heat by convection at tip: $\dot q_{cond.} \rightarrow | \rightarrow \dot q_{conv}$ $\qquad \dot q_{cond.} = \dot q_{conv.}$

$$-kA\left.\frac{dT}{dx}\right|_{x=L} = hA(T_L - T_\infty)$$

General Solution to Gov. Eq:

Define $\theta = T - T_\infty \rightarrow \frac{d\theta}{dx} = \frac{dT}{dx}, \quad \frac{d^2\theta}{dx^2} = \frac{d^2T}{dx^2}$

$$\rightarrow \frac{d^2\theta}{dx^2} - \frac{hP}{KA}\theta = 0$$

Define $m^2 = \frac{hP}{KA} \longrightarrow \boxed{\frac{d^2\theta}{dx^2} - m^2\theta = 0}$

Try $\theta = \sin(mx), \quad \frac{d\theta}{dx} = m\cos(mx), \quad \frac{d^2\theta}{dx^2} = -m^2\sin(mx)$

Try $\theta = e^{mx}, \quad \frac{d\theta}{dx} = me^{mx}, \quad \frac{d^2\theta}{dx^2} = m^2 e^{mx}$

gen. sol.: $\boxed{\theta = C_1 e^{mx} + C_2 e^{mx}}$

Case 1) Assume fin is infinitely long: $T \rightarrow T_\infty$ @ $x \rightarrow \infty$

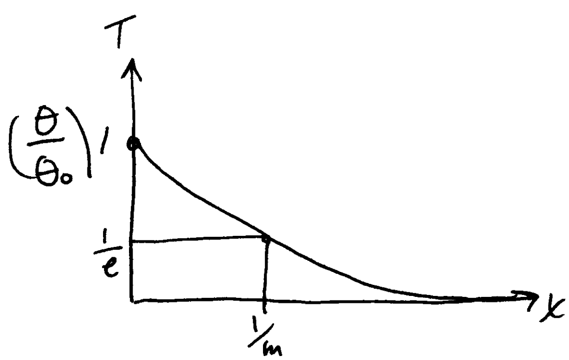
$$T = T_\infty \rightarrow \theta = T - T_\infty = 0$$

$$0 = C_1 e^{m\infty} + \cancel{C_2 e^{-m\infty}}^{\rightarrow 0} \qquad C_1 = 0$$

@ $x = 0$, $T = T_w \rightarrow \theta_0 = T_w - T_\infty$

$$\theta_0 = C_2 e^{-m \times 0} \rightarrow C_2 = \theta_0$$

$\rightarrow \theta = \theta_0 e^{-mx}$ or $\boxed{\frac{\theta}{\theta_0} = e^{-mx}}$ non-dimensional



Case 2) Tip is insulated : $\left.\frac{dT}{dx}\right|_{x=L} = 0$, $T = T_w$ @ $x = 0$

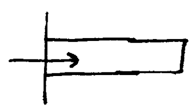
- solve for c_1, c_2

$$\boxed{\frac{\theta}{\theta_0}} = \frac{e^{-mx}}{1+e^{-2mL}} + \frac{e^{mx}}{1+e^{2mL}} = \boxed{\frac{\cosh(m(L-x))}{\cosh(mL)}}$$

Case 3) convection at tip : $\left.KA\frac{dT}{dx}\right|_{x=L} = hA(T-T_\infty)$

$$\frac{T-T_w}{T_w-T_\infty} = \boxed{\frac{\theta}{\theta_0} = \frac{\cosh m(L-x) + (h/mK)\sinh m(L-x)}{\cosh mL + \frac{h}{mK}\sinh mL}}$$

$$\dot{q}_0 = -KA\left.\frac{dT}{dx}\right|_{x=0}$$

$\dot{q}_0$ 

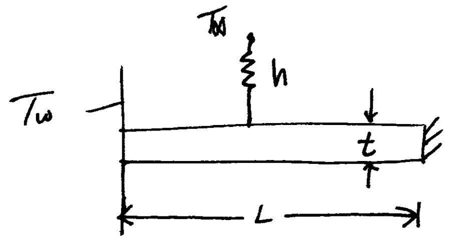
<u>To find heat flux :</u> compute $\dot{q}_0 = -KA\left.\frac{dT}{dx}\right|_{x=0}$

B.C. 1 : $\dot{q}_0 = \theta_0\sqrt{hPKA}$

B.C. 2 : $\dot{q}_0 = \sqrt{hPKA}\,\theta_0\tanh(mL)$

B.C. 3 : $\dot{q}_0 = \sqrt{hPKA}\,(T_w - T_\infty)\dfrac{\sinh(mL) + h/mK\cosh(mL)}{\cosh(mL) + \frac{h}{mK}\sinh(mL)}$

ex) Aluminum Fin: $K = 200\ \frac{W}{m\text{-}K}$, 3 mm thick, 7.65 mm long, rect. c.s.
infinitely ~~thin~~ deep, $T_\infty = 50°C$, $h = 10\ \frac{W}{m^2\text{-}K}$, $T_W = 300°C$



– case 2

$z \to \infty,\ 2t \to 0$

$$m = \sqrt{\frac{hP}{KA}} = \sqrt{\frac{h(2z + \cancel{2t})}{k(zt)}} = \sqrt{\frac{h(2z)}{k(zt)}} = \sqrt{\frac{2h}{kt}} = 5.77\ \frac{1}{m}$$

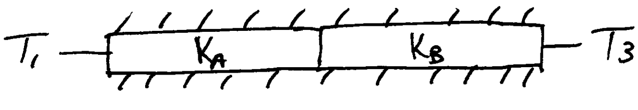
$$\dot{q}_0 = \tanh(mL)\sqrt{hPKA}\,\theta_0 \qquad hPKA = h(2z + \cancel{2t})k(zt) = 2hkz^2t$$

$$\frac{\dot{q}_0}{z} = \dot{q}_0' = \tanh(mL)\sqrt{2hkt}\,\theta_0 \longrightarrow \underline{\underline{\dot{q}_0' = 360\ \frac{W}{m}}}$$

discussion 9/29/0

Sect. 3.1.4 – contact resistance

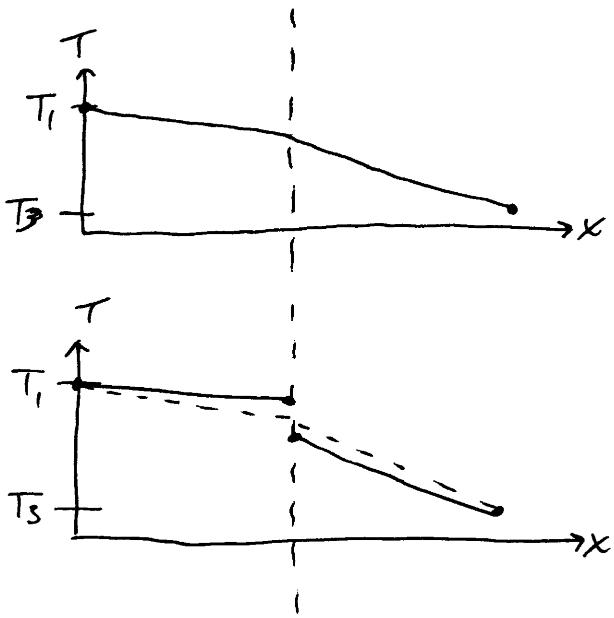
two rods in contact with each other



$k_A > k_B$

$T_1 > T_3$

$\dot{q}''' = 0$, s.s.



– Ideal temp. profile

– Actual temp. profile

– Jump in temp. due to contact resistance.

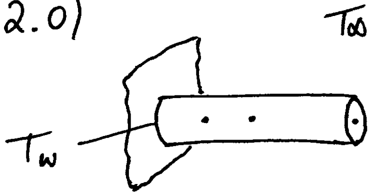
$$\dot{q}'' = k_A \frac{T_1 - T_{2A}}{\Delta x_A} = k_B \frac{T_{2B} - T_3}{\Delta x_B} = \frac{T_{2A} - T_{2B}}{1/h_c}$$

$\frac{1}{h_c}$ = contact resistance

$\frac{1}{h_c} = R''_{t,c}$ (book)

$\left[\frac{m^2\text{-}K}{W}\right]$

3.1.2.0)



$L = 100\ mm$ $\quad$ $T_w = 200°C$ $\quad$ $T_{25mm} = ?$

$d = 5\ mm$ $\quad$ $T_\infty = 20°C$ $\quad$ $T_{50mm} = ?$

$K = 133\ \frac{W}{m\text{-}K}$ $\quad$ $h = 30\ \frac{W}{m^2\text{-}K}$ $\quad$ $T_{100\,mm} = ?$

$$m = \sqrt{\frac{hP}{KA_c}} = \sqrt{\frac{h\pi d}{K\frac{\pi}{4}d^2}} = 13.43\ m^{-1}$$

$$\frac{h}{mK} = 0.0168$$

$$\frac{\theta}{\theta_o} = \frac{\cosh m(L-x) + \left(\frac{h}{mK}\right)\sinh m(L-x)}{\cosh mL + \frac{h}{mK}\sinh mL}$$

$\theta = T - T_\infty$

$\theta_o = T_w - T_\infty$

$$T(x) = \frac{\cosh m(L-x) + \left(\frac{h}{mK}\right)\sinh m(L-x)}{\cosh mL + \frac{h}{mK}\sinh mL}\left(T_w - T_\infty\right) + T_\infty$$

3.124) Two long copper rods soldered together

$d = 10\,mm$

Melting $Pt_{solder} = 650°C$

$T_\infty = 25°C$

$h = 10 \, \frac{W}{m^2\text{-}K}$

$K = 379 \, \frac{W}{m\text{-}K}$

$P_{min} = ?$

T_∞

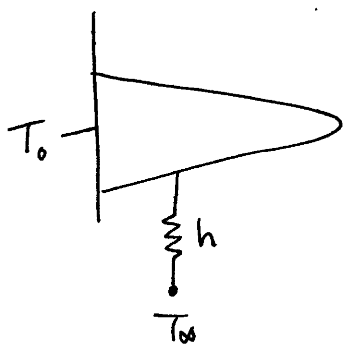
heat flux: $q_f = \sqrt{hPKA_c}\,(T - T_\infty) \times 2$ rods

$$= 2\sqrt{h \pi d K \frac{\pi}{4} d^2}\,(T - T_\infty) = 120.9 \text{ W}$$

9/30/09

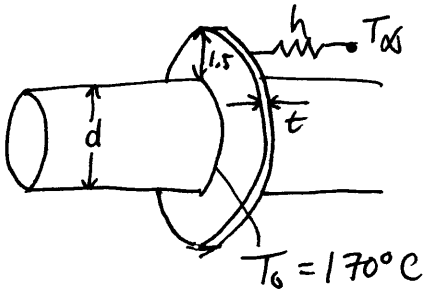
Fin Efficiency:

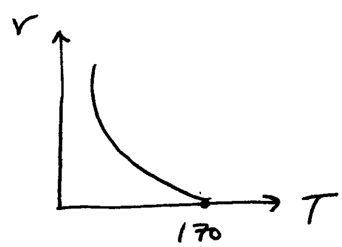
$$\eta_{fin} = \frac{\text{actual heat transfer}}{\text{maximum possible H.T. by fin}} = \frac{\dot{q}_f}{hA_f(T_o - T_\infty)}$$



η values – Fig 3.18, 3.19, Table 3.5

ex) Aluminum fin 1.5 cm wide, 1.0 mm thick, on a 2.5 cm diameter tube. Tube surface at 170°C, $T_\infty = 25°C$, $h = 130\ \frac{W}{m^2\text{-}K}$, $K_{Al} = 200\ \frac{W}{m\text{-}K}$. H.T. per fin?





$$\boxed{\dot{q}_{max} = hA_f(T_o - T_\infty)} = \left(130\ \frac{W}{m^2\text{-}K}\right)\left[2\pi(r_o^2 - r_i^2) + 2\pi r_o t\right](170°C - 25°C)$$

To find η_f, fig. 3.19

$$r_{2c} = r_2 + \frac{t}{2} = r_o + \frac{t}{2}$$

$$L_c = L + \frac{t}{2} = (r_o - r_i) + \frac{t}{2}$$

$$L_c^{3/2}\left(\frac{h}{kA_p}\right)^{1/2} \quad A_p = L_c t$$

$$L_c = L + \frac{t}{2} = 1.5\,cm + \frac{1\,mm}{2} = 0.0155\,m$$

$$r_{2c} = r_i + L_c = 1.25\,cm + 1.55\,cm = 0.0280\,m$$

$$A_p = t(r_{2c} - r_i) = (1\times10^{-3})(0.028 - 0.0125) = 1.55\times10^{-5}\,m^2$$

$$L_c^{3/2}\left(\frac{h}{kA_p}\right) = 0.395$$

$$\frac{r_{2c}}{r_i} = 2.24$$

From 3.19, $\eta_f = 0.82$

$$\eta_{effectiveness} = \frac{\text{heat transfer w/ fin}}{\text{heat transfer w/o fin}} = \frac{\eta_f A_f h (T_0 - T_\infty)}{h A_b (T_0 - T_\infty)}$$

Chapter 4 10/5/09

2-D Conduction

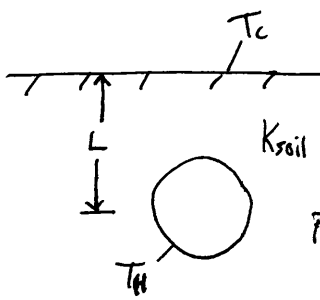
Conduction shape factor:

Applicable when the temperature of the body & surroundings are specified

$$\boxed{\dot{q} = k S \Delta T_{overall}}$$

k — thermal conductivity of medium

S — conduction shape factor Table 4-1



Pipe buried in ground

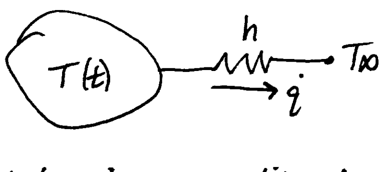
Unsteady Heat Conduction

Temperatures can change w/ time

- Lumped capacity (Uniform temperature through-out, but changing w/ time)
- semi-infinite solid
- multi-dimensional conduction

Lumped Capacity

- Temperature in body is spatially uniform, but changing w/ time.



Heat loss by convection results in temp drop in body.

$$\dot{q} = h A_s (T(t) - T_\infty) = -m c_p \frac{dT}{dt} = -\rho V c_p \frac{dT}{dt}$$

B.C.: $T = T_i$ @ $t = 0$

Separate Variables:

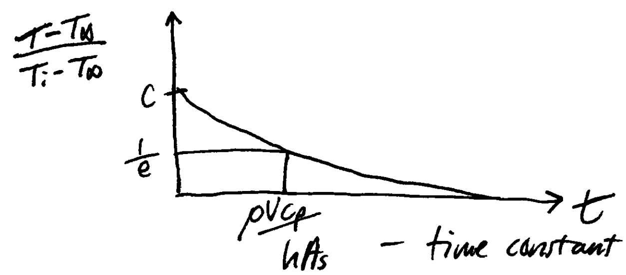
$$\frac{dT}{T - T_\infty} = -\frac{hA_s}{mc_p}\,dt$$

$$\ln(T - T_\infty) = -\frac{hA_s}{\rho V c_p} t + C_1$$

$$T - T_\infty = \exp\left(-\frac{hA_s}{\rho V c_p} t\right) C_2$$

I.C.: $T = T_i$ @ $t = 0 \longrightarrow C_2 = T_i - T_\infty$

$$\boxed{\frac{T - T_\infty}{T_i - T_\infty} = \exp\left[-\frac{hA_s}{\rho V c_p} t\right]}$$



- If conduction in body ~~is~~ large, R_{th} in body is small and can be neglected, so Temp is uniform

$$q'' \sim k\frac{T_i - T_\infty}{L} \sim h(T_o - T_\infty)$$

$$\rightarrow T_i - T_o \sim \frac{hL}{k}(T_o - T_\infty + T_i - T_i)$$

$$\sim B_i(T_o - T_i) + B_i(T_i - T_\infty)$$

$$\rightarrow \frac{T_i - T_o}{T_i - T_\infty} = \frac{B_i}{1 + B_i}$$ - For lumped capacity, $B_i << 1$

- For most geometries, use $L = \frac{V}{A_s}$. If $B < 0.1$, solutions will be within 5%

Example

Steel ball bearing: $c_p = 0.46 \frac{kJ}{\ }$, $k = 35 \frac{W}{m\text{-}K}$, $D = 5\,mm$, $T_i = 450°C$, $h = 10 \frac{W}{m^2\text{-}K}$, $T_\infty = 100°$

How long until $T = 150°C$?

$$B_i = \frac{h(V/A_s)}{k} = \frac{h\left(\frac{4}{3}\pi r^3 / 4\pi r^2\right)}{k} = \frac{h(r/3)}{k} \longrightarrow B_i = 0.0024 < 0.1 \quad \therefore \text{lumped capacity}$$

$$\frac{T - T_\infty}{T_i - T_\infty} = \exp\left(\frac{-hA_s}{\rho c_p V} t\right), \quad \frac{hA_s}{\rho V c_p} = 3.34 \times 10^{-4}\,s^{-1}$$

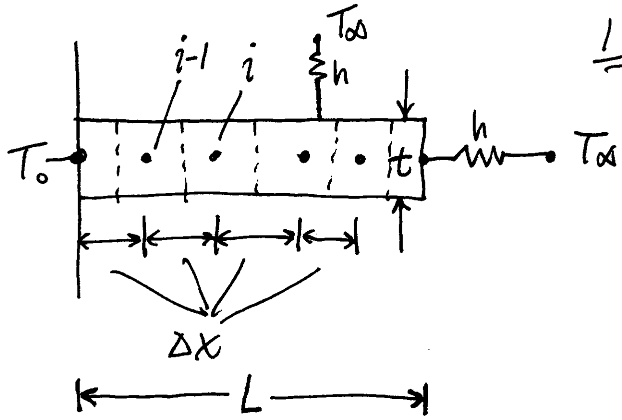
$$\frac{T - T_\infty}{T_i - T_\infty} = 0.143 = \exp(-3.34E{-}4\,t)$$

$$\boxed{t = 5826\,s}$$

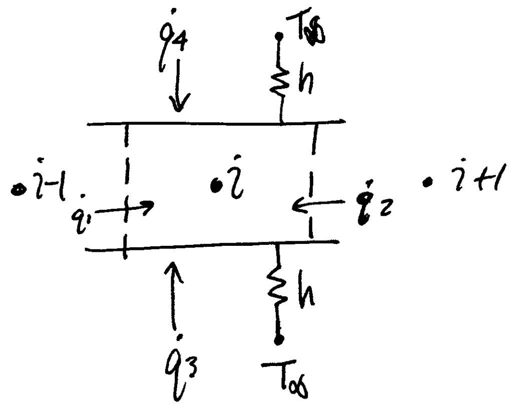
discussion 10/6/09

Numerical Method for Heat Conduction

1-D Fin



1) Assign nodes on fin
2) Draw control forces around each node that are equally spaced between nodes
3) Perform energy balance on each node



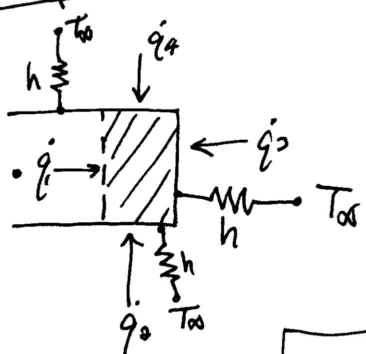
Energy balance on node i: $\dot{q}_1+\dot{q}_2+\dot{q}_3+\dot{q}_4=0$

$$\underbrace{kt\frac{T_{i-1}-T_i}{\Delta x}}_{\dot{q}_1}+\underbrace{kt\frac{T_{i+1}-T_i}{\Delta x}}_{\dot{q}_2}+\underbrace{2h\Delta x(T_\infty-T_i)}_{\dot{q}_3+\dot{q}_4}=0$$

Solve for T_i: $T_i\left[2\frac{kt}{\Delta x}+2h\Delta x\right]=\frac{kt}{\Delta x}(T_{i-1}+T_{i+1})+2h\Delta x T_\infty$

$$\rightarrow \boxed{T_i=\frac{\frac{kt}{\Delta x}(T_{i-1}+T_{i+1})+2h\Delta x T_\infty}{2\frac{kt}{\Delta x}+2h\Delta x}}$$ node eq. for node i

At tip:



$$\Sigma \dot{q}_i=0$$

$$kt\frac{T_{n-1}-T_n}{\Delta x}+h\frac{\Delta x}{2}(T_\infty-T_n)+h\frac{\Delta x}{2}(T_\infty-T_n)+ht(T_\infty-T_n)=0$$

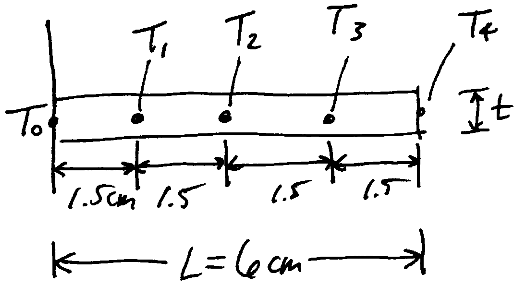
Solve for T_n:

$$\boxed{T_n=\frac{\frac{kt}{\Delta x}T_{n-1}+htT_\infty+h\Delta x T_\infty}{\frac{kt}{\Delta x}+ht+\Delta x h}}$$

Can solve these eq.s to get T_i's using Gauss-Seidal

1) Guess values for T_i
2) Use node eqs to update T_i's.
3) Iterate until convergence

ex)



$T_0 = 100°C$ $\quad k = 170 \frac{W}{m\cdot K}$

$\Delta x = 1.5cm$ $\quad h = 200 \frac{W}{m^2 \cdot K}$

$t = 0.002m$ $\quad T_\infty = 0$

Node eqs for interior nodes T_1, T_2, T_3:

$$T_i = \frac{\frac{kt}{\Delta x}(T_{i-1} + T_{i+1})}{\frac{2kt}{\Delta x} + 2h\Delta x} = \underline{\underline{0.442(T_{i-1} + T_{i+1})}}$$

Node eqn at tip:

$$T_n = \frac{\frac{kt}{\Delta x} T_{n-1}}{\frac{kt}{\Delta x} + ht + \Delta x h} \longrightarrow \underline{\underline{T_n = 0.869\, T_{n-1}}}$$

Gauss-Seidel: $\quad T_\infty = 0$

1) Assume $T_1 = T_2 = T_3 = T_4 = 0$

2) $T_1 = 0.442(100 + \underbrace{0}_{T_2 = 0}) = 44.2$

$T_2 = 0.442(T_1 + T_3) = 0.442(44.2 + 0) = 19.5$

$T_3 = 0.442(\underbrace{T_2}_{T_2 = 19.5} + \underbrace{T_4}_{T_4 = 0}) = 8.64$

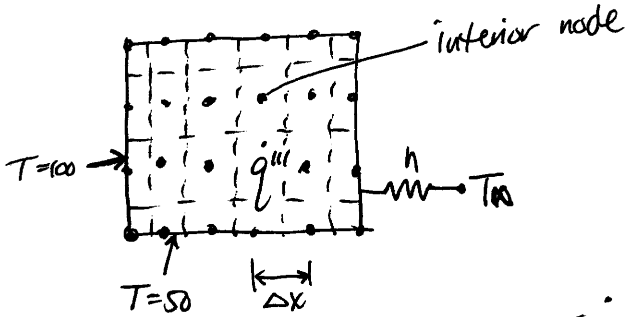
$T_4 = 0.869 T_3 = 7.50$

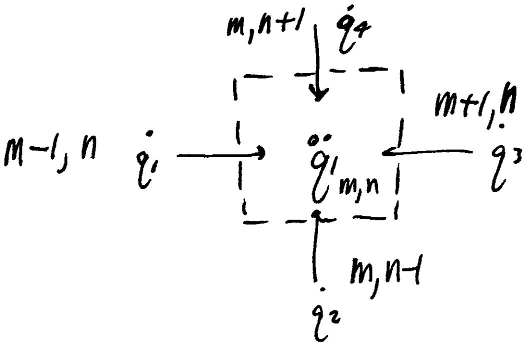
$\underbrace{\qquad\qquad\qquad\qquad}_{1^{st}\ \text{Iteration}}$

Excel:

T_0	T_1	T_2	T_3	T_4
100	0	0	0	0

2-D Conduction:



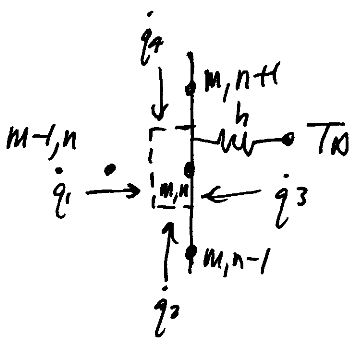


$$\sum \dot{q}_i = 0$$

$$k\Delta y \frac{T_{m-1,n} - T_{m,n}}{\Delta x} + k\Delta x \frac{T_{m,n-1} - T_{m,n}}{\Delta y} + k\Delta y \frac{T_{m+1,n} - T_{m,n}}{\Delta x}$$

$$+ k\Delta x \frac{T_{m,n+1} - T_{m,n}}{\Delta y}$$

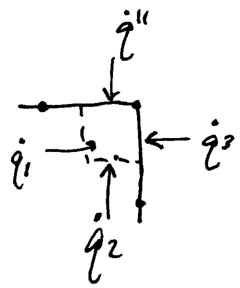
$$+ \dot{q}''' \Delta x \Delta y = 0$$



$$\sum \dot{q}_i = 0$$

$$k\Delta y \frac{T_{m-1,n} - T_{m,n}}{\Delta x} + k\frac{\Delta x}{2} \frac{T_{m,n-1} - T_{m,n}}{\Delta y} + h\Delta y \left(T_\infty - T_{m,n}\right) + k\frac{\Delta x}{2} \frac{T_{m,n+1} - T_{m,n}}{\Delta y}$$

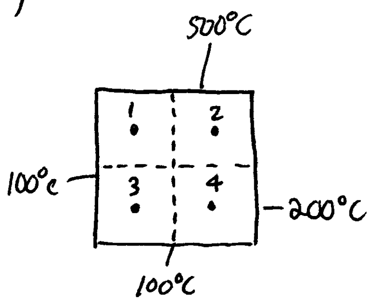
$$+ \dot{q}''' \frac{\Delta x}{2} \Delta y = 0$$



$$\sum \dot{q}_i = 0$$

$$k\frac{\Delta y}{2} \frac{T_{m-1,n} - T_{m,n}}{\Delta x} + k\frac{\Delta x}{2} \frac{T_{m,n-1} - T_{m,n}}{\Delta y} + h\frac{\Delta y}{2} \left(T_\infty - T_{m,n}\right) + \dot{q}'' \frac{\Delta x}{2} + \dot{q}''' \frac{\Delta x \Delta y}{4} =$$

ex)



$\Delta x = \Delta y$

$\dot{q}''' = 0$

n	T_1	T_2	T_3	T_4
1	0	0	0	0
2	150	212.5	87.5	150
3	225	268.75	147	178
⋮	⋮	⋮	⋮	⋮
n	263	288	163	188

$$T_1(n=2) = \frac{500°C + 100°C + 0°C + 0°C}{4}$$

3.122) Aluminum Alloy Fin $T_b = 100°C$ $T_\infty = 25°C$ $k = 180 \frac{W}{m \cdot K}$ $h = 100 \frac{W}{m^2 \cdot K}$ $L = 10\,mm$ $t = 1\,mm$ $w \gg t$

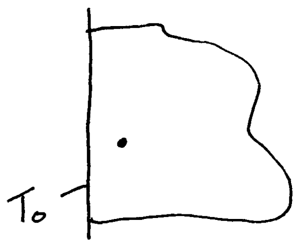
a) Find $\dot{q}'_f$, η_f, ε_f, $R'_{th,f}$ for Case A: convection at tip
Case B: $\dot{q}''|_{x=L} = 0$

Chapter 5

10/7/09

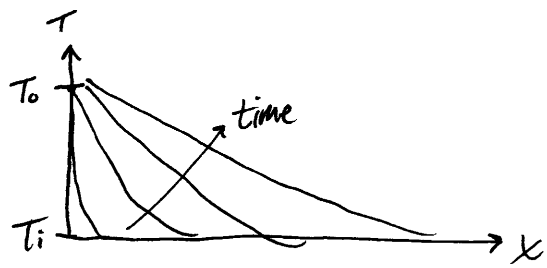
Transient Heat Flow in a Semi-Infinite Solid

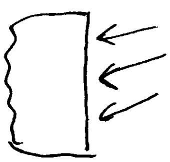
– ground



$T = T_i$ initially everywhere in solid

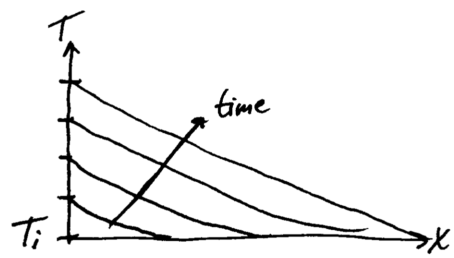
$T = T_o$ at surface at $t = 0$

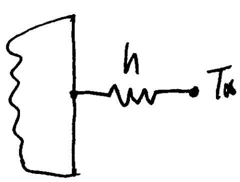




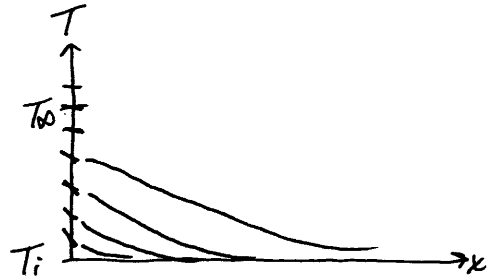
$T = T_i$ initially

$$\dot{q}_o'' = -k \left.\frac{dT}{dx}\right|_{x=0}$$





$$\dot{q}_o'' = h(T_\infty - T_i) = -k \left.\frac{dT}{dx}\right|_{x=0}$$



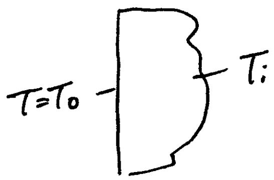
Gov. Eq:

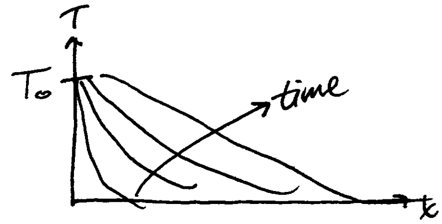
$$\frac{d}{dx}\left(k\frac{dT}{dx}\right) + \cancelto{0}{\frac{d}{dy}\left(k\frac{dT}{dy}\right)} + \cancelto{0}{\frac{d}{dz}\left(k\frac{dT}{dz}\right)} + \cancelto{0}{\dot{q}'''} = \rho c_p \frac{dT}{dt}$$

$$\frac{d}{dx}\left(k\frac{dT}{dx}\right) = \rho c_p \frac{dT}{dt} \longrightarrow \frac{d^2T}{dx^2} = \frac{1}{k/\rho c_p}\frac{dT}{dt}$$

$$\boxed{\frac{d^2T}{dx^2} = \frac{1}{\alpha}\frac{dT}{dt}} \qquad \alpha = \frac{k}{\rho c_p}$$

For $T=T_0$ at surface for $t>0$:





I.C.: $T(x,0)=T_i$

B.C.: $T(0,t)=T_0$

Dimensional Analysis: $\eta = \dfrac{x}{\sqrt{4\alpha t}}$

check to see if η reduces partial D.E. to O.D.E.

$$\frac{dT}{dx} = \frac{dT}{d\eta}\frac{d\eta}{dx} = \frac{dT}{d\eta}\frac{1}{\sqrt{4\alpha t}}$$

$$\frac{d^2T}{dx^2} = \frac{d}{dx}\left(\frac{dT}{dx}\right) = \frac{d}{d\eta}\left(\frac{dT}{dx}\right)\frac{d\eta}{dx} = \frac{d}{d\eta}\left(\frac{dT}{d\eta}\frac{d\eta}{dx}\right)\frac{d\eta}{dx}$$

$$= \left(\frac{d^2T}{d\eta^2}\frac{d\eta}{dx} + \frac{dT}{d\eta}\frac{d^2\eta}{d\eta\, dx}\right)\frac{d\eta}{dx} = \frac{d^2T}{d\eta^2}\left(\frac{d\eta}{dx}\right)^2$$

$$\longrightarrow \boxed{\frac{d^2T}{d\eta^2} = -2\eta\frac{dT}{d\eta}} \quad \text{– O.D.E.} \quad T=T(\eta)$$

B.C. $T(0,t)=T_0 \longrightarrow T(0)=T_0$

I.C. $T(x,0)=T_i \longrightarrow T(\infty)=T_i$

$$\boxed{\frac{T-T_0}{T_i-T_0} = \frac{2}{\sqrt{\pi}}\int_0^{\eta}\exp(-\beta^2)\,d\beta = \operatorname{erf}(\eta)}$$

– error function

– values of $\operatorname{erf}(\eta)$ in Appendix B.2

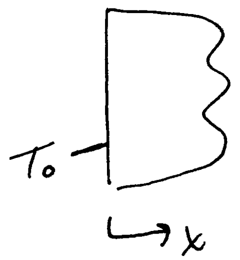
Heat Flux: $q'' = -k\dfrac{T_i-T_0}{\sqrt{\pi\alpha t}}\exp\left(-\dfrac{x^2}{4\alpha t}\right)$

@ surface: (x=0) $\boxed{q_0'' = \dfrac{(T_0-T_i)k}{\sqrt{\pi\alpha t}}}$

Constant heat flux at surface:

$$T - T_i = \frac{2\dot{q}_o''\sqrt{\alpha t/\pi}}{k}\exp\left(\frac{-x^2}{4\alpha t}\right) - \frac{\dot{q}_o'' x}{k}\left(1 - \operatorname{erf}\frac{x}{2\sqrt{\alpha t}}\right)$$

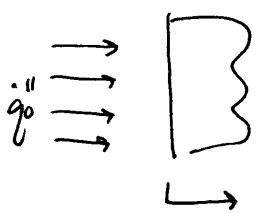
10/12/09



$$\frac{T-T_0}{T_i-T_0} = erf(\eta)$$

$$\dot{q}_x = -KA\frac{dT}{dx} \rightarrow \dot{q}_0 = \frac{KA(T_0-T_i)}{\sqrt{\pi\alpha t}}$$

Semi-Infinite Solid, $\dot{q}_0'' = const$ at surface



$$T-T_i = \frac{2\dot{q}_0''\sqrt{\alpha t/\pi}}{K}\exp\left(-\frac{x^3}{4\alpha t}\right)$$
$$-\frac{\dot{q}_0'' x}{K}\left(L-erf\frac{x}{2\sqrt{\alpha t}}\right)$$

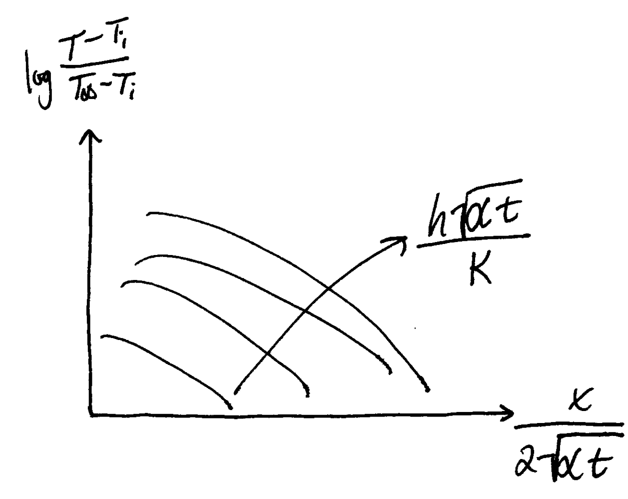
Convective B.C. at surface: $T_\infty \overset{h}{\sim}$

$$h(T_\infty-T)\Big|_{x=0} = -K\frac{dT}{dx}\Big|_{x=0}$$

Temp. Profile $T(x,t)$ is given by:

$$\frac{T-T_i}{T_\infty-T_i} = 1-erf X - \left[\exp\left(\frac{hx}{K}+\frac{h^2\alpha t}{K^2}\right)\right]\left[1-erf\left(X+\frac{h\sqrt{\alpha t}}{K}\right)\right]$$ — don't use

→ use a graph instead



EXAM #1
Mon, Oct. 26

ex) Large block of steel : $K = 45 \frac{W}{m-K}$ $\quad T_o = 35°C$

$\alpha = \frac{K}{\rho c_p} = 1.4 \times 10^{-5} \, m^2/s \quad T_s = 250°C$

Initially at T_o, the surface raised to T_s. What is T at a depth of 2.5 cm after 30s?

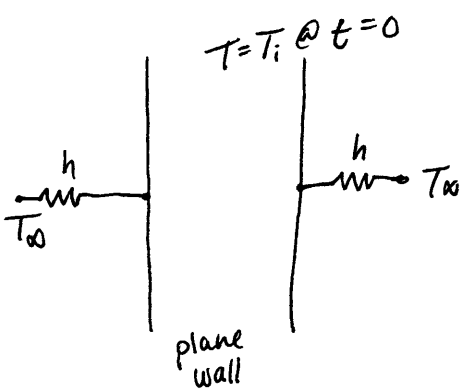
$$\frac{T - T_o}{T_i - T_o} = erf\left(\frac{x}{2\sqrt{\alpha t}}\right) \qquad \eta = \frac{x}{2\sqrt{\alpha t}} = \frac{0.025 \, m}{2\sqrt{1.4 \times 10^{-5} \frac{m^2}{s} \times 30 \, s}} = 0.61$$

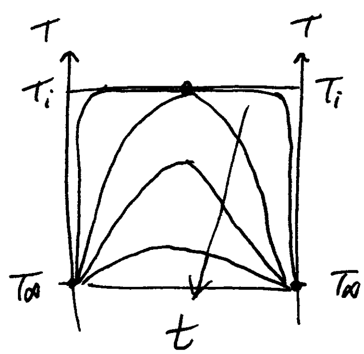
Appendix B. 2

$$erf(\eta) = erf(0.61) = 0.612 \longrightarrow T = 118.4 °C$$

Transient Conduction (Finite Geometries)

- Plane walls, cylinders, spheres





Use Heisler Charts for temp. profiles in solid at any time & location of solid.

Chart 1 : gives T at centerline as a fcn of time

Chart 2 : gives T of centerline but based on centerline T

Def: $\theta = T(x,t) - T_\infty$ or $T(r,t) - T_\infty$

$\theta_i = T_i - T_\infty$ - initial temp. diff.

$\theta_o = T_o - T_\infty$ - centerline ~~temp~~

chart 1: θ_o / θ_i — non-dimensional centerline T

chart 2: θ / θ_o — off-center T

To get T in solid off-center:

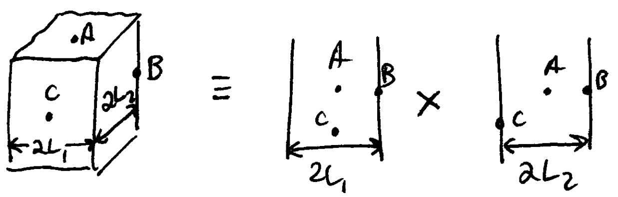
$$\frac{\theta}{\theta_i} = \underbrace{\left(\frac{\theta_o}{\theta_i}\right)}_{\text{chrt. 1}} \times \underbrace{\left(\frac{\theta}{\theta_o}\right)}_{\text{chrt. 2}} = \frac{T - T_\infty}{T_i - T_\infty}$$

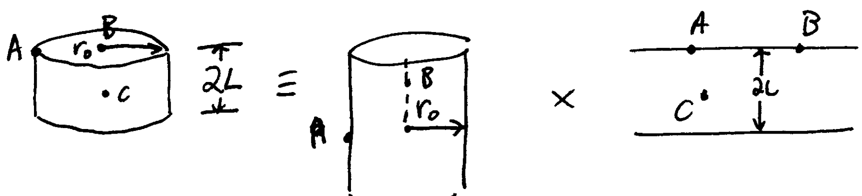
10/19/09

Multi-Dimensional Systems

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Assume $T(x,z,t) = T_1(x,t)\; T_2(z,t)$
where T_1 & T_2 are solutions to corresponding 1-D problems





Heat transferred from body

Intersection of two bodies:

$$\left(\frac{Q}{Q_o}\right)_{tot} = \left(\frac{Q}{Q_o}\right)_1 + \left(\frac{Q}{Q_o}\right)_2 \left[1 - \left(\frac{Q}{Q_o}\right)_1\right]$$

$\left(\frac{Q}{Q_o}\right)_1$ = heat lost for corresponding 1-D body (body 1)

$\left(\frac{Q}{Q_o}\right)_2$ = " "" " " " (body 2)

Intersection of three bodies:

$$\left(\frac{Q}{Q_o}\right)_{tot} = \left(\frac{Q}{Q_o}\right)_1 + \left(\frac{Q}{Q_o}\right)_2 \left[1 - \left(\frac{Q}{Q_o}\right)_1\right] + \left(\frac{Q}{Q_o}\right)_3 \left[1 - \left(\frac{Q}{Q_o}\right)_1\right]\left[1 - \left(\frac{Q}{Q_o}\right)_2\right]$$

discussion 10/20/09

5.60) Sphere: $D = 30\,mm$ $\quad k = 1.7\ \frac{W}{m\text{-}K}$ $\quad h = 75\ \frac{W}{m^2\text{-}K}$

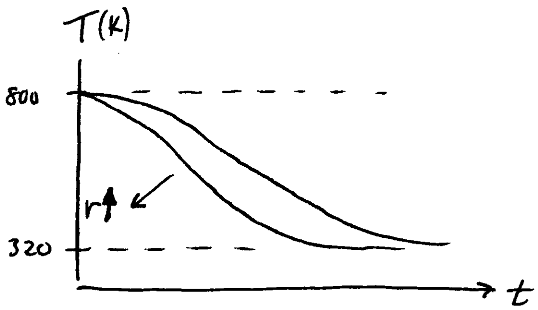
$T_i = 800\,K$ $\quad T_\infty = 320\,K$ $\quad \alpha = \frac{k}{\rho c_p}$

$\rho = 400\ kg/m^3$ $\quad c_p = 1600\ J/kg\text{-}K$

a) T @ center and surface as a fcn of time:

$Bi = \frac{hL}{k}$ L - characteristic length $\quad L = \frac{V}{A_s}$ ← general rule

sphere: $Bi = \frac{h\left(\frac{r_o}{3}\right)}{k} = \frac{75\left(\frac{0.015}{3}\right)}{1.7} = 0.22$ ∴ lumped capacitance



$T(r,t) = 415$ $\quad r = r_o$ $\quad t = ?$

b) time required for the surface to be 415 K:

$r/r_o = 1$

$\cancel{\frac{\theta}{\theta_o} = \frac{T(r,t) - T_\infty}{T_o - T_\infty} = \frac{415 - 320}{800 - 320} = 0.198}$

$Bi = \frac{hr_o}{k} = \frac{75 \times 0.015}{1.7} = 0.66 \rightarrow Bi^{-1} = 1.51 \xrightarrow{\text{charts}} \frac{\theta}{\theta_o} = 0.72$

$\frac{\theta}{\theta_i} = \frac{T(r,t) - T_\infty}{T_i - T_\infty} = \frac{415 - 320}{800 - 320} = 0.198 = \frac{\theta_o}{\theta_i} \times \frac{\theta}{\theta_o} = \frac{\theta_o}{\theta_i} \times 0.72$

$\frac{\theta_o}{\theta_i} = \frac{0.198}{0.72}$ $\cancel{= \frac{T_o - T_\infty}{T_i - T_\infty} = \frac{T_o - 320}{800 - 320} \rightarrow T_o = 452\,K}$ $\quad \frac{\theta_o}{\theta_i} = 0.275$

$Fo = \frac{\alpha t}{r_o^2} \rightarrow t = \frac{Fo\, r_o^2}{\alpha} = \frac{Fo\, r_o^2}{k/\rho c_p} = \boxed{74\ sec}$

c) heat flux at surface at time from part b:

$$q'' = h\left(T(r_o, \overset{74s}{t}) - T_\infty\right) = 75\,(415 - 320) = \boxed{7125\ \frac{W}{m^2}}$$

d) energy lost to cool surface to 415 K:

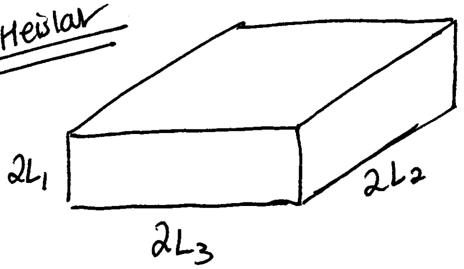
$$Bi^2 Fo = \frac{h^2 \alpha t}{k^2} = \frac{75^2 \left(\frac{1.7}{400 \times 1600}\right) \times 74s}{1.7^2} = 0.383$$

$$\frac{Q}{Q_o} = 0.75$$

$$Q_o = m C_p \Delta T = \rho V C_p \Delta T = 400 \times \left(\frac{4}{3}\pi \times 0.015^3\right) \times 1600 \times (800 - 320) = 4343\ J$$

$$\therefore\ Q = 0.75 \times Q_o = 0.75 \times 4343 = \boxed{3257\ J}$$

3-D Heisler



$2L_1 = 0.06\,m$
$2L_2 = 0.09\,m$
$2L_3 = 0.2\,m$

$T_\infty = 313\ K$
$h = 50\ \frac{W}{m^2\text{-}K}$ $\quad K = 1\ \frac{W}{m\text{-}K}$
$T_i = 1600\,K$ $\quad \rho = 2{,}050\ \frac{Kg}{m}$
$t = 50\ min = 3000s$ $\quad \alpha = 0.51 \times 10^{-6}\ m$

a) T at center & corner after t.

CENTER:

L_1 // $Bi^{-1} = \frac{K}{hL} = \frac{1}{50 \times 0.03} = 0.667$, $F_o = \frac{\alpha t}{L^2} = \frac{0.51 \times 10^{-6} \times 3000}{0.03^2} = 1.7$, $P_1(0,t) = 0.22$

L_2 // $Bi^{-1} = 0.44$, $F_o = 0.756$
$P_2(0,t) = 0.5$

L_3 // $Bi^{-1} = 0.2$, $F_o = 0.153$
$P_3(0,t) = 0.85$

$$\frac{T(0,0,0,t) - T_\infty}{T_i - T_\infty} = P_1(0,t) \times P_2(0,t) \times P_3(0,t) = \boxed{433\ K}$$

CORNER:

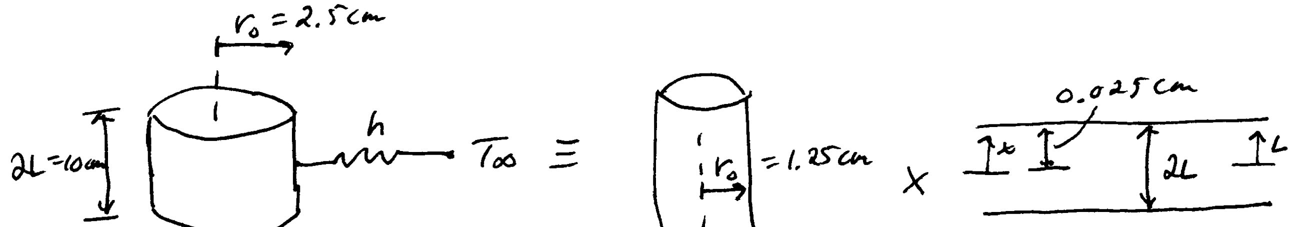
$$P(L_1,t) = P(0,t) \times \frac{\theta(L_1,t)}{\theta_o}$$

$$\frac{T(L_1,L_2,L_3,t) - T_\infty}{T_i - T_\infty} = P_o(L_1,t) \times P(L_2,t) \times P(L_3,t)$$
$$= (0.22 \times 0.55)(0.5 \times 0.43)(0.85 \times 0.25) = 0.00553$$

$$\frac{\theta(L_1,t)}{\theta_o} = 0.55,\quad \frac{\theta(L_2,t)}{\theta_o} = 0.43,\quad \frac{\theta(L_3,t)}{\theta_o} = 0.25$$

$$\boxed{T(L_1,L_2,L_3,t) = 320\,K}$$

ex) Short Aluminum cylinder 5.0 cm in diameter, 10 cm long is initially at 200°C. It is suddenly subject to $T_\infty = 70°C$ & $h = 525 \frac{W}{m^2\text{-}K}$. What is T @ $r = 1.25$ cm and a distance 0.025 cm from one end of the cylinder 1 minute after exposure? What is the heat lost?



$L = 5$ cm, $x = 5\text{ cm} - 0.025\text{ cm}$
$= 4.375$ cm

Infinite Cylinder : Heisler Charts

$$\frac{r}{r_o} = \frac{1.25\text{ cm}}{2.50\text{ cm}} = 0.5, \quad \frac{k}{hr_o} = 16.38, \quad \frac{\alpha t}{r_o^2} = 8.064$$

From charts: $\frac{\theta_o}{\theta_i} = 0.38 \quad \frac{\theta}{\theta_o} = 0.98 \qquad \alpha = \frac{k}{\rho c_p}$

$$\left(\frac{\theta}{\theta_i}\right)_{\substack{\text{inf.}\\ \text{cyl.}}} = \frac{\theta_o}{\theta_i}\frac{\theta}{\theta_o} = 0.372$$

Infinite Plate : $L = 5$ cm, $x = 4.375$ cm Heisler Charts

$$\frac{x}{L} = 0.875, \quad \frac{k}{hL} = 8.19, \quad \frac{\alpha t}{L^2} = 202$$

$$\frac{\theta_o}{\theta_i} = 0.75, \quad \frac{\theta}{\theta_o} = 0.95 \rightarrow \left(\frac{\theta}{\theta_o}\right)_{\text{inf. plate}} = 0.7125$$

Short Cylinder: $\left(\frac{\theta}{\theta_i}\right)_{\substack{\text{short}\\ \text{cyl.}}} = \left(\frac{\theta}{\theta_i}\right)_{\substack{\text{inf}\\ \text{cyl}}} \times \left(\frac{\theta}{\theta_i}\right)_{\text{inf plate}} = 0.372 \times 0.7125 = 0.265$

$$\frac{T - T_\infty}{T_i - T_\infty} = 0.265 = \frac{T - 70°C}{200 - 70} \Rightarrow T = 105°C$$

Heat Lost from short Cylinder:

Plate : $\frac{hL}{k} = 0.122, \quad \frac{h^2 \alpha t}{k^2} = 0.03 \rightarrow \frac{Q}{Q_o} = 0.22$

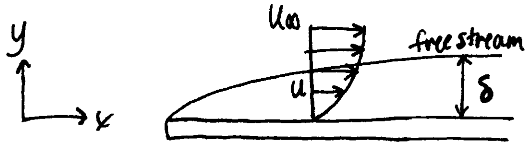
Cylinder : $\frac{hr_o}{k} = 0.061, \quad \frac{Q}{Q_o} = 0.55$

$$\left(\frac{Q}{Q_o}\right)_{\substack{\text{short}\\ \text{cylinder}}} = 0.22 + 0.55\,(1 - 0.22) = 0.649$$

$Q_o = m c_p \Delta T_{max} = \rho V c_p (T_i - T_\infty) = 62\text{ KJ} \rightarrow Q = (0.649)(62\text{ KJ}) = 40\text{ KJ}$ in 1 min

Chapter 6 10/21/09

Convection – Heat Transfer from fluid motion (forced convection, natural convection)

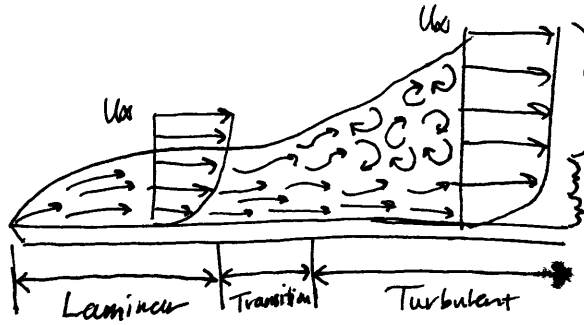


$u=0$ at wall $\rightarrow$ conduction

if U_∞ increases, δ decreases, conduction increases

$$\dot{q}''_w = h\left(T_w - T_\infty\right) = -K_f \left.\frac{\partial T}{\partial y}\right|_{y=0}$$

Laminar & Turbulent Flow



B.L. transition affected by:

- free stream turb
- bend in wall
- surface roughness
- streamline acceleration
- suction & blowing

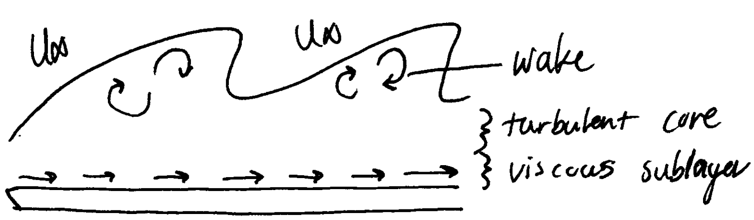
Reynold's Number

$$Re_x = \frac{\rho U_\infty x}{\mu} = \frac{U_\infty x}{\nu}$$

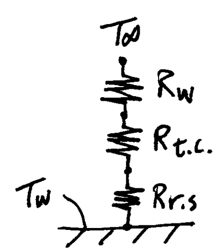
$3\times10^5 < Re_x < 5\times10^5$ $\quad Re_{x,c} \equiv 5\times10^5$

For pipe flow transition to turbulence: $Re_d = \frac{VD}{\nu} \approx 2{,}300$

Turbulent B.C.



Viscous sublayer $\rightarrow$ ~1% of B.L. thickness
- velocities are very small

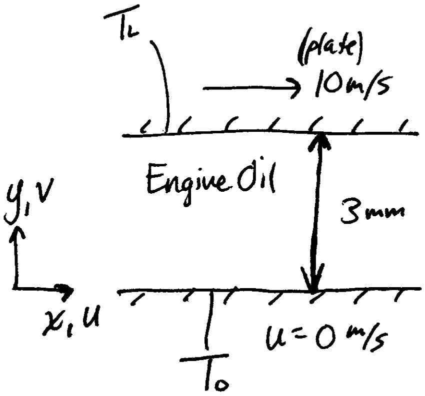


Turbulent Core $\rightarrow$ A large range of eddy sizes
~10% of B.L. thickness

Turbulent Wake $\rightarrow$ ~90% of B.L. thickness

discussion 10/27/09

Fluid Mechanics & Heat Transfer



$\rho = 888 \ kg/m^3$

$k = 0.145 \ \frac{W}{m\text{-}K}$

$\nu = 900 \times 10^{-6} \ m^2/s$

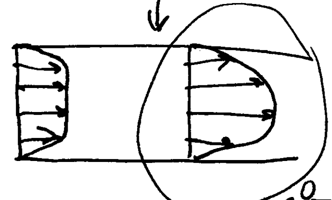
$\mu = 0.799 \ \frac{N\text{-}s}{m^2}$

steady state

$\dot{q}''' = 0$

1-D

fully developed



Velocity profile:

body force

$$\rho\left(\cancelto{0}{u\frac{\partial u}{\partial x}} + \cancelto{0}{v\frac{\partial u}{\partial y}}\right) = \cancelto{0}{-\frac{\partial P}{\partial x}} + \frac{\partial}{\partial x}\left\{\mu\left[\cancelto{0}{2\frac{\partial u}{\partial x}} - \frac{2}{3}\cancelto{0}{\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)}\right]\right\} + \frac{\partial}{\partial y}\left[\mu\left(\frac{\partial u}{\partial y} + \cancelto{0}{\frac{\partial v}{\partial x}}\right)\right] + \cancelto{0}{X}$$

$$\mu \frac{d^2u}{dy^2} = 0 \longrightarrow \frac{d^2u}{dy^2} = 0 \longrightarrow \frac{du}{dy} = C_1 \rightarrow u = C_1 y + C_2$$

B.C.: $u = 0 \ @ \ y = 0$, $u = V \ @ \ y = L$

$$u = \frac{y}{L} V = \frac{y \,(10 \ m/s)}{0.003 \ m}$$

Temperature Profile:

Energy Equation: Reduces to $\boxed{0 = k\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2}$

$\frac{\partial u}{\partial y} = \frac{V}{L}$ – velocity profile

$$k\frac{\partial^2 T}{\partial y^2} = -\mu\left(\frac{V}{L}\right)^2 \longrightarrow \frac{dT}{dy} = -\frac{\mu}{k}\left(\frac{V}{L}\right)^2 y + C_1 \rightarrow T = -\frac{\mu}{k}\left(\frac{V}{L}\right)^2 y^2 + C_1 y + C_2$$

B.C.: $T = T_0 \ @ \ y = 0$, $T = T_L \ @ \ y = L$

solve for C_1, C_2 & substitute

$$T(y) = T_0 + \frac{\mu}{2k} V^2 \left[\frac{y}{L} - \left(\frac{y}{L}\right)^2\right] + (T_L - T_0)\frac{y}{L}$$

where does maximum occur?

$$\text{set } \frac{dT}{dy} = 0 \rightarrow y = \frac{\left(\frac{T_L - T_0}{L}\right) + \left(\frac{\mu}{2k}\frac{V^2}{L}\right)}{\frac{\mu}{k}\left(\frac{V}{L}\right)^2}$$

$y = 1.61 \ mm$

$T_{max} = 89.3°C$

Heat Fluxes:

$$\dot{q}_0'' = -k \frac{dT}{dy}\Big|_{y=0} \quad , \quad \dot{q}_L'' = -k \frac{dT}{dy}\Big|_{y=L}$$

$$\frac{dT}{dy} = -\frac{\mu}{k}\left(\frac{V}{L}\right)^2 y^2 + \frac{T_L - T_0}{L} + \frac{\mu}{2k}\frac{V^2}{L}$$

$$\dot{q}_0'' = \left[\left(\frac{T_L - T_0}{L}\right) + \frac{\mu}{2k}\frac{V^2}{L}\right](-k) = -14.3 \ \mathrm{kW/m^2}$$

$$\dot{q}_L'' = 12.3 \ \mathrm{kW/m^2}$$

11/2/09

$$Re_x = \frac{U_\infty x}{\nu} \qquad \text{momentum: } C_{f,x} = \frac{0.664}{\sqrt{Re_x}}$$

$$\nu = \mu/\rho$$

$Pr \sim 1$ $\quad T_w = \text{const.}, \ \dot{q}'' = \text{const}$

$$Nu_x = \frac{hx}{k} = 0.332 Pr^{1/3} (Re_x)^{1/2}\left[1 - \left(\frac{x_0}{x}\right)^{3/4}\right]^{-1/3}$$

x_0 - unheated starting length

To get avg. h from $0 \to L$: $\bar{h}_L = \frac{1}{L}\int_0^L h_x \, dx$

For $x_0 = 0$, $\overline{Nu}_L = \frac{\bar{h}_L L}{k} = 0.664 Pr^{1/3} (Re_L)^{1/2}$ — Laminar

For $Pr \neq 1$: oils, liquid metals

$$Nu_x = \frac{0.464 \, Re_x^{1/2} Pr^{1/3}}{\left[1 + \left(\frac{0.0207}{Pr}\right)^{2/3}\right]^{1/4}} \quad - \ \dot{q}_0'' = \text{const.}$$

Staton Number: $St_x = \dfrac{h_x}{\rho C_p U_\infty}$

Laminar to Turbulent transition at: $Re_{crit} = 5\times10^5$

$\overline{C}_f = \dfrac{0.455}{\log Re_L}2.584 - \dfrac{A}{Re_L}$, $Re_L < 10^9$ A depends on R_{crit}

$St_x Pr^{2/3} = 0.0296\, Re_x^{-1/5}$. $5\times10^5 < Re < 10^7$

$St_x Pr^{2/3} = 0.185\,(\log Re_x)^{-2.584}$ $10^7 < Re_x < 10^9$

$\overline{St}\; Pr^{2/3} = 0.037\, Re_L^{-1/5} - 871\, Re_L^{-1}$ $R_{crit} = 5\times10^5$

---

ex) flat plate width 1 m, $T_w = 230°C$, series of strip heaters each 50 mm long
$T_\infty = 25°C$, $U_\infty = 60$ m/s. Which heater has highest power requirement?

Transition occurs at: $Re_{crit} = 5\times10^5$

$5\times10^5 = \dfrac{U_\infty x_{cr}}{\nu} = \dfrac{U_\infty x_{cr}}{\mu/\rho} \longrightarrow x_{cr} = 0.22$ m μ-value from book

$\rho = \dfrac{P}{RT}$ μ, c_p, k - not affected by gas pressure

$\alpha = \dfrac{k}{\rho c_p}$ - dependent on pressure

$Pr = \dfrac{\nu}{\alpha} = \dfrac{\mu/\rho}{k/\rho c_p} = \dfrac{\mu c_p}{k}$

$T_{film} = \dfrac{T_w + T_\infty}{2} = 127°C$

Heater 1

compute $\overline{h}_L$ for laminar flow between 0-5 cm

$\overline{Nu}_L = 0.664\, Re_L^{1/2}\, Pr^{1/3} = \dfrac{\overline{h}_L L}{k_{f1}}$

$= 0.664\left(\dfrac{60\text{ m/s}\times 0.05\text{ m}}{\nu_{air}}\right)^{1/2}(0.7)^{1/3} = 198 = \dfrac{\overline{h}_L(0.05\text{ m})}{0.0338}$

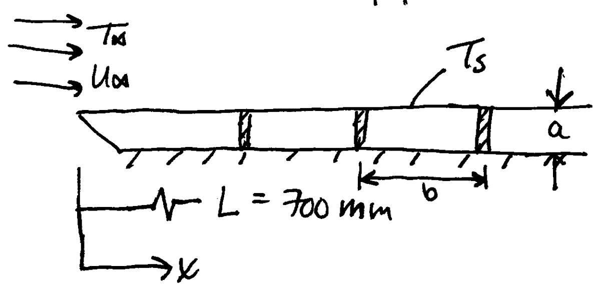
$\therefore \overline{h}_L = 134\ \dfrac{W}{m^2 K}$

$\therefore q_{conv} = 134\ \dfrac{W}{m^2K}(0.05\text{ m}\times 1\text{ m})(230-25) = 1370$ W

discussion 11/3/09

7.8) Flat plate of width 1m, maintained at a uniform surface temp. $T_s = 150°C$ w/ heat-generating rectangular modules of thickness $a = 10$ mm, length $b = 50$ mm. Each block is insulated from its neighbors. Atmospheric air at 25°C flows over plate at $U_\infty = 30$ m/s. $K = 5.2$ W/m-K, $C_p = 320$ J/kg-K, $\rho = 2{,}300$ kg/m³

module properties



$\nu = 22.02 \times 10^{-6}$ m²/s

$Pr = 0.698$

a) Find required power generation $\dot{q}'''$ in a module 700 mm from leading edge:

$$q = \dot{q}''' \times V$$

$$\bar{h} A (T_s - T_\infty) = \dot{q}''' \times A \times a$$

$$Re_{L+b/2} = \frac{U_\infty (L + b/2)}{\nu} = \frac{30 \text{ m/s} \left(0.7\text{m} + \frac{0.05}{2}\text{m}\right)}{22.02 \times 10^{-6} \text{ m}^2/\text{s}}$$

$\therefore Re_{L+b/2} = 9.877 \times 10^5$ – turbulent flow ($> 5 \times 10^5$)

<u>Turbulent Flow, T_s = const.</u>

$$Nu_x = 0.0296\, Re_x^{4/5}\, Pr^{1/3} = 0.0296 \left(9.877 \times 10^5\right)^{4/5} (0.698)^{1/3} = 1640$$

$$Nu = \frac{hL}{K} \longrightarrow 1640 = \frac{hL}{K} = \frac{h (L + b/2)}{K_{air}} = h \frac{\left(0.7 + \frac{0.05}{2}\right)\text{m}}{0.0308 \frac{\text{W}}{\text{m-K}}}$$

$$\therefore h = 69.7 \frac{\text{W}}{\text{m}^2\text{-K}} \qquad \therefore \dot{q}''' = \frac{h (T_s - T_\infty)}{a} = \boxed{8.713 \times 10^5 \text{ W/m}^3}$$

b) Max temp. in module:

$$T_{max} = \frac{\dot{q}''' a^2}{2K} + T_s = \boxed{158.4°C}$$

7.9) An electric air heater consists of a horizontal array of thin metal strips 10 mm long. Air flow is parallel w/ length of plate, strips are 0.2 m wide, 25 strips, $U_\infty = 2\ m/s$

During operation, each strip is at 500°C and $T_\infty = 25°C$

a) What is the rate of convection heat transfer from the:

First strip?

$$Re_L = 5\times10^5 = \frac{U_\infty X_{cr}}{\nu} \rightarrow X_{cr} = (5\times10^5)(43.5\times10^{-6}\ \tfrac{m^2}{s})\left(\tfrac{1}{2\ m/s}\right) = 10.9\ m$$

$10.9 > (0.2\times25\ \text{strips})$

∴ All air flow is laminar

Isothermal & Laminar

$$\overline{Nu}_x = 0.664\ Re_x^{1/2}\ Pr^{1/3}$$

$$\overline{h}_x = \frac{k\,\overline{Nu}_x}{x} = \frac{0.0429\ \frac{W}{m\cdot K}}{0.01\ m} \times \left(0.664\left(\frac{2\ m/s \times 0.01\ m}{43.54\times10^{-6}\ m^2/s}\right)^{1/2}(0.683)^{1/3}\right) = 53.8\ \frac{W}{m^2 K}$$

$$q_1 = hA(T_s - T_\infty) = 53.8\ \frac{W}{m^2 K}(0.01\ m \times 0.2\ m)(500-25)°C = \boxed{51.1\ W}$$

Fifth Strip?

$$q = q_{0-5} - q_{0-4} = \overline{h}_{0-5}\underbrace{(5\times0.01\ m \times 0.2\ m)}_{A_s\ 0-5}(T_s - T_\infty) - \overline{h}_{0-4}\underbrace{(4\times0.01\ m\times0.2\ m)}_{A_s\ 0-4}(T_s - T_\infty)$$

$$= \boxed{12.2\ W}$$

7.10] Air at $T_\infty = 25°C$ and $u_\infty = 25\ m/s$ flowing over both surfaces of a 1 m long flat plate maintained at 125°C. What is the rate of heat transfer per unit width from the plate for Re values of 10^5, 5×10^5, and 10^6?

$$Re_L = \frac{u_\infty L}{\nu} = \frac{25\ m/s \times 1\ m}{20.72 \times 10^{-6}\ m^2/s} = 1.21 \times 10^6$$

Eq. 7.38: $\overline{Nu}_L = \left(0.037\, Re_L^{4/5} - A\right) Pr^{1/3}$ $\qquad A = 0.037\, Re_{x,cr}^{4/5} - 0.664\, Re_{x,cr}^{1/2}$

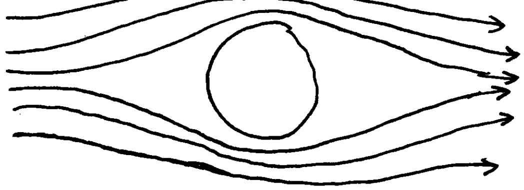
$\underline{\underline{10^5?}}$

$$Re_{x,cr} = 10^5 \longrightarrow A = 0.037 (10^5)^{4/5} - 0.664 (10^5)^{1/2} =$$

$Re_{x,cr}$	10^5	5×10^5	10^6
A	160	871	1671
$\overline{Nu}_L$	2272	1641	931
$\bar{h}_L\ [W/m^2K]$	67.9	49.1	27.8
$\dot{q}\ [W/m]$	13,580	9820	5560

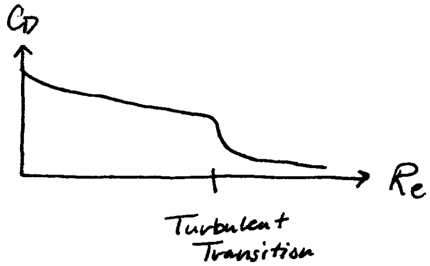
11/4/09

Cylinder in Cross Flow



Drag:

$$F_D = C_D A_f \frac{\rho u_\infty^2}{2}$$



Churchill & Bernstein:

$$Nu = 0.3 + \frac{0.62\, Re_D^{1/2} Pr^{1/3}}{\left[1 + \left(\frac{0.4}{Pr}\right)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{Re}{282{,}000}\right)^{5/8}\right]^{4/5}$$

$$Nu = c\left(\frac{u_\infty d}{\nu_f}\right)^m Pr_f^{1/3}$$

c, m depend on Re

Table 7.2

Sphere:

$$Nu = \frac{hd}{k} = 0.37\,(Re_D)^{0.6}$$ gases: $17 < Re_D < 70{,}000$ McAdams

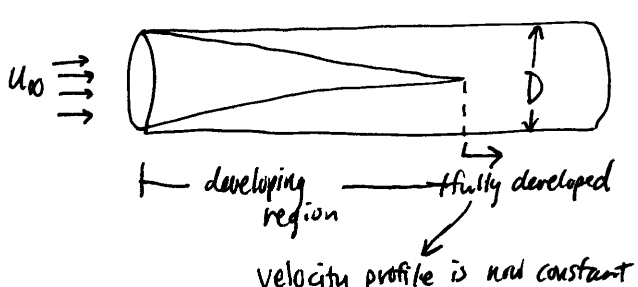
$$Nu_d\, Pr_f^{-0.3} = 0.97 + 0.68\,(Re_D)^{0.5}$$ liquids: $1 < Re_D < 2000$ Kramers

$$Nu_d = 2 + \left(0.4\, Re_D^{1/2} + 0.06\, Re_D^{2/3}\right) Pr^{0.4} \left(\frac{\mu_{air}}{\mu_s}\right)^{1/4}$$ $3.5 < Re < 8 \times 10^4$, $0.7 < Pr < 380$ Whitaker

11/9/09

Internal Flow

Flow in circular pipe:



case: flow is fully developed, constant properties

$$\rho u \cancelto{0}{\frac{\partial u}{\partial x}} + \rho \cancelto{0}{v_r} \frac{\partial u}{\partial r} + \frac{dP}{dx} = \frac{1}{r}\frac{\partial}{\partial r}\left(r \mu \frac{\partial u}{\partial r}\right)$$

B.C.: $r = 0,\ \left.\frac{du}{dr}\right|_{r=0} = 0$ (symmetry)

$r = r_0,\ u = 0$ (no-slip B.C.)

Integrate: $d\left(r\mu \frac{du}{dr}\right) = r \frac{dP}{dx} dr$

$$r\mu \frac{du}{dr} = \frac{r^2}{2}\frac{dP}{dx} + C_1$$

using symm. B.C.: $C_1 = 0$

$$du = \frac{r}{2\mu}\frac{dP}{dx} dr$$ integrate: $u = \frac{r^2}{4\mu}\frac{dP}{dx} + C_2$

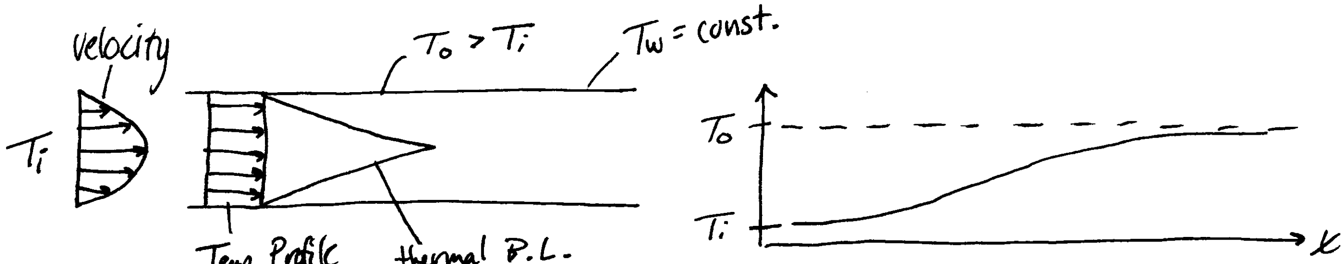
using no-slip B.C.: $0 = \frac{r_0^2}{4\mu}\frac{dP}{dx} + C_2$

$$\therefore \boxed{u = \frac{r_0^2}{4\mu}\left(-\frac{dP}{dx}\right)\left(1 - \frac{r^2}{r_0^2}\right)}$$

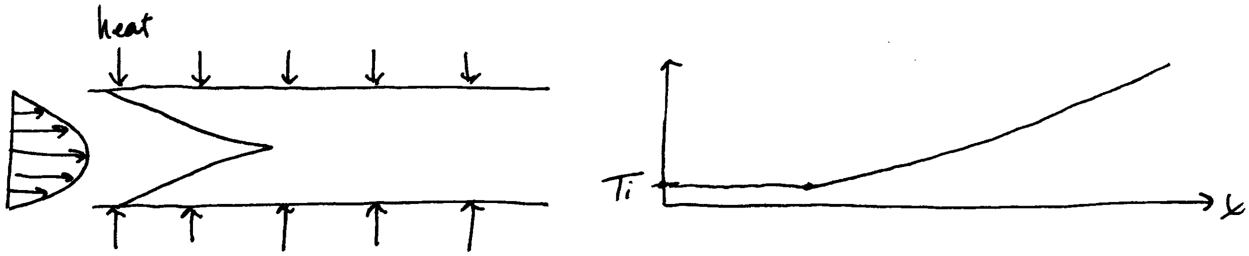
u_{max} occurs at $r = r_0$

$$u_{max} = \frac{r_0^2}{4\mu}\left(-\frac{dP}{dx}\right)$$

Heat Transfer in Pipes: fully developed



($T(x) = T_i$ until out of thermal B.L.)



Temp Profile for case: T_w = const.

Energy: $\rho u c_p \frac{\partial t}{\partial x} + \cancel{v_r}^{0} \rho c_p \frac{\partial t}{\partial r} - \frac{k}{r}\frac{\partial}{\partial r}\left(r\frac{\partial t}{\partial r}\right) + \cancel{\frac{k}{r^2}\frac{\partial^2 t}{\partial \theta^2}}^{0} = 0$

$$\boxed{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial t}{\partial r}\right) = \frac{u}{\alpha}\frac{\partial t}{\partial x}}$$

$$\boxed{\begin{array}{ll} \text{B.C.:} & r = 0, \frac{\partial t}{\partial r} = 0 \quad \text{(symmetric)} \\ & r = r_0, T = T_0 \end{array}}$$

we know $u(r)$, so:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial t}{\partial r}\right) = \frac{u_0}{\alpha}\left(1 - \frac{r^2}{r_0^2}\right)\frac{\partial t}{\partial x}, \quad u_0 = u_{max}$$

$$\rightarrow \partial\left(r\frac{\partial t}{\partial r}\right) = \frac{u_0}{\alpha}\left(r - \frac{r^3}{r_0^2}\right)\frac{\partial t}{\partial x}(\partial r)$$

integrate $\rightarrow$ $r\frac{\partial t}{\partial r} = \frac{u_0}{\alpha}\left(\frac{r^2}{2} - \frac{r^4}{4r_0^2}\right)\frac{\partial t}{\partial x} + C_1$

symmetry B.C.: $C_1 = 0$

divide by r, integrate, 2nd B.C.:

$$\boxed{t - t_0 = \frac{1}{\alpha}\frac{\partial T}{\partial x}\frac{u_0 r_0^2}{4}\left[\left(\frac{r}{r_0}\right)^2 - \frac{1}{4}\left(\frac{r}{r_0}\right)^4\right]}$$

Heat Flux: $q'' = h(T_w - T_b)$

$T_b \triangleq$ bulk temp. / "mixing cup" temp.

$$T_b = \frac{\int_0^{r_0} t u 2\pi r dr}{\int_0^{r_0} u 2\pi r dr}$$

substitute $t(r)$ & $u(r)$:

$$\boxed{t_b = t_c + \frac{7}{96}\frac{u_0 r_0^2}{\alpha}\frac{\partial T}{\partial x}}$$

$$\boxed{t_w = t_c + \frac{3}{16}\frac{u_0 r_0^2}{\alpha}\frac{\partial T}{\partial x}}$$

$$q_0 = hA\left(T_w - T_b\right) = kA\left.\frac{\partial T}{\partial r}\right|_{r=r_0}$$

$$\boxed{\frac{\partial T}{\partial r} = \frac{u_0 r_0}{4\alpha}\frac{\partial t}{\partial x}}$$

$$h = \frac{k\left.\frac{\partial T}{\partial r}\right|_{r=r_0}}{T_w - T_b} = \frac{48}{11}\frac{k}{d_0}$$

d_0 - pipe diameter

Nusselt #: $$\boxed{Nu = \frac{h d_0}{k} = \frac{48}{11} = 4.36}$$

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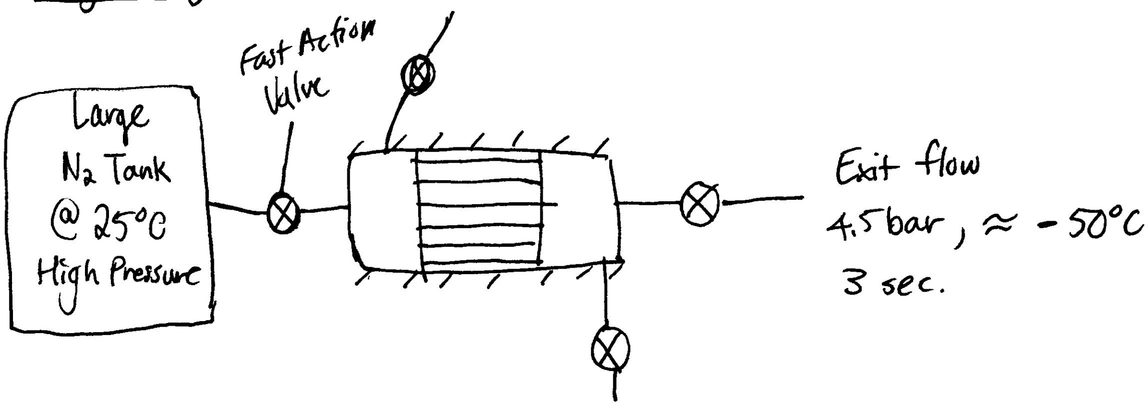
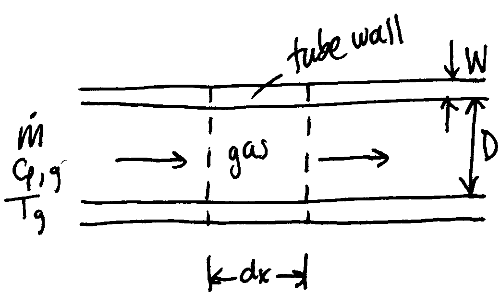
discussion 11/10/09

Design Project:



Analysis of Tube Bundle



Energy Balance on Gas

$$dq = \dot{m}\, c_{p,g}\frac{\partial T_g}{\partial x}dx = h\pi D\, dx\left(T_w - T_g\right)$$

$$\rightarrow \boxed{\frac{\partial T_g(x,t)}{\partial x} = \frac{h\pi D}{\dot{m} c_{p,g}}\left[T_w(x,t) - T_g(x,t)\right]}$$

Energy Balance on tubewall

$$\dot{q} = m c_p \frac{\partial T_w}{\partial t} = W\pi D\, dx \times \rho_w\, c_p \frac{\partial T_w}{\partial t} = h\pi D\, dx\left(T_g - T_w\right)$$

$$\rightarrow \boxed{\frac{\partial T_w}{\partial t} = -\frac{h}{W\rho_w c_{p,w}}\left[T_w(x,t) - T_g(x,t)\right]}$$

Non-Dimensionalized

$$\theta_g = \frac{T_g - T_{g,i}}{T_{w,i} - T_{g,i}}, \quad \theta_w = \frac{T_w - T_{g,i}}{T_{w,i} - T_{g,i}}$$

Then:

$$\frac{\partial \theta_g}{\partial x^*} = (\theta_w - \theta_g)$$

$$\frac{\partial \theta_w}{\partial t^*} = -(\theta_w - \theta_g)$$

where $x^* = \frac{x}{\lambda}$, $\lambda = \frac{\dot{m} C_{p,g}}{h \pi D}$

$t^* = \frac{t}{\tau}$, $\tau = \frac{w \rho C_{p,w}}{h}$

$$\Delta T_g = \frac{1}{\lambda}\left[T_w(x,t) - T_g(x,t)\right]\Delta x$$

$$\rightarrow \boxed{T_{g,x+\Delta x} = \frac{1}{\lambda}\left[T_w(x,t) - T_g(x,t)\right]\Delta x + T_{g,x}}$$

$$\Delta T_w = -\frac{1}{\tau}\left[T_w(x,t) - T_g(x,t)\right]\Delta t$$

$$\rightarrow \boxed{T_{w,t+\Delta t} = -\frac{1}{\tau}\left[T_w(x,t) - T_g(x,t)\right]\Delta t + T_{w,t}}$$

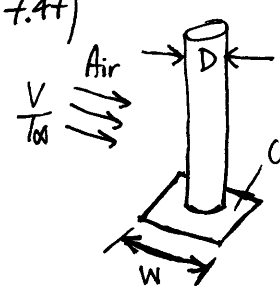
Solve for $T_g(x,t)$, $T_w(x,t)$ Excel

	$t=0$	$t=0$	$t=\Delta t$	$t=\Delta t$
x	$T_{w,i}$	T_g	$T_w(x,t)$	$T_g(x,t)$
0	-50	25		
Δx	-50			
$2\Delta x$	-50			
$3\Delta x$	-50			
$4\Delta x$				

Parameters that can be changed

Cost \$ $\leftarrow$ (D, L, w, # of tubes, material, $T_{w,i}$)

7.47)



$w = 4$ mm $\quad V = 10$ m/s

$L = 12$ mm $\quad T_\infty = 300$ K

$D = 2$ mm $\quad T_b = 350$ K

Copper
$k = 398 \frac{W}{m \cdot K}$

$$T_f = \frac{T_\infty + T_b}{2} = 325\ K$$

a) Average convection coefficient for the surface of the pin?

$\nu = 18.41 \times 10^{-6}$ m²/s

$Pr = \frac{\nu}{\alpha} = 0.704$

$$Re = \frac{VL}{\nu} = \frac{VD}{\nu} = \frac{10\ m/s \times 0.002\ m}{18.41 \times 10^{-6}\ m^2/s} = 1087 \therefore \text{Laminar}$$

$$\text{Eq. 7.54: } \overline{Nu}_D = 0.3 + \frac{0.62 Re_D^{1/2} Pr^{1/3}}{\left[1 + \left(\frac{0.4}{Pr}\right)^{2/3}\right]^{1/4}} \times \left[1 + \left(\frac{Re_D}{282000}\right)^{5/8}\right]^{4/5} = 16.7 = \frac{\bar{h} D}{k} \rightarrow \boxed{\bar{h} = 235 \frac{W}{m^2 \cdot K}}$$

b) assuming the convection coefficient at the tip to equal value from (a), find pin heat transfer rate

$$M = \sqrt{\bar{h} P K A_c}\ \Theta_b = \sqrt{\bar{h} \pi D K \times \frac{\pi D^2}{4}}\ [T_b - T_\infty] = 2.15\,W$$

$$m = \left(\frac{\bar{h} P}{K A_c}\right)^{1/2} = 34.3\ m^{-1}$$

$$q_f = M \frac{\sinh(mL) + \frac{\bar{h}}{mK}\cosh(mL)}{\cosh(mL) + \frac{\bar{h}}{mK}\sinh(mL)} = 0.868\ W$$

$$\dot{q}_T = \dot{q}_f + \dot{q}_b = 0.868\ W + \bar{h}\left(w^2 - \frac{\pi D^2}{4}\right)(T_b - T_\infty) = 1.019\,W$$

7.53) Aluminum transition line, $D = 20\,mm$, $R'_{elec} = 2.636 \times 10^{-4}\ \Omega/m$, $I = 700\,A$

Insulate the line, but this increases conductor operating temp.

a) $T_\infty = 20°C$, $v = 10\ m/s$. Find conductor temp.

$$\dot{q}''' = \frac{i^2 R'_{elec}}{A_c} = \frac{(700A)^2 (2.636 \times 10^{-4}\ \Omega/m)}{\frac{\pi}{4}(0.02\ m)^2} = 4.111 \times 10^5\ W/m^3$$

$$\dot{q}''' \times [A_c \times L] = h A_s \Delta T \rightarrow \dot{q}''' \times \left(\frac{\pi}{4} D^2 \times L\right) = h\,(\pi D L)(T_s - T_\infty)$$

$T_c = 40°C \rightarrow T_f = 30°C$ $\boxed{T_c = 45.8°}$

11/11/09

Internal Flow con't

If $T_w \neq$ const.: $Nu = 4.364$ (fully developed flow)

If $T_w =$ const.: $Nu = 3.66$ (also fully developed)

Developing Flow

Laminar

Fig. 8.10: a) local Nu b) average Nu (Laminar)

Turbulent

$$Nu_D = \frac{hD}{k} = 0.023\, Re_D^{4/5}\, Pr^{1/3} \quad \text{- Colburn Eq.}$$

Dittus-Boelter Eq.: $\boxed{Nu_D = 0.023\, Re_D^{4/5}\, Pr^n}$, $n = 0.4$ for heating $T_s > T_m$

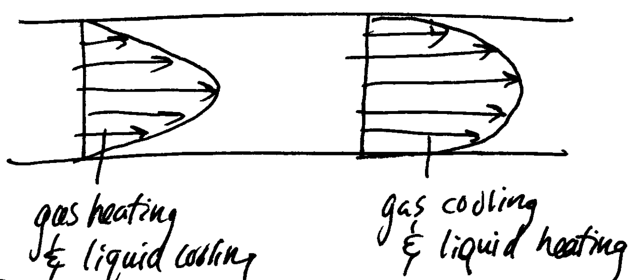
$n = 0.3$ for cooling $T_s < T_m$

for $0.6 \lesssim Pr \lesssim 160$

Seider-Tate: take viscosity into account

$\mu_{gas} \uparrow \quad T \uparrow$

$\mu_{liquid} \downarrow \quad T \uparrow$



$$\boxed{Nu_d = 0.027\, Re_D^{0.8}\, Pr^{1/3} \left(\frac{\mu}{\mu_w}\right)^{0.14}}$$

$0.7 < Pr < 16{,}700$

$Re_D \gtrsim 10{,}000$, $\frac{L}{D} \geq 10$

Gnielinski: $$\boxed{Nu_D = \frac{(f/8)(Re_D - 1000)\,Pr}{1 + 12.7\,(f/8)^{1/2}\,(Pr^{2/3} - 1)}}$$

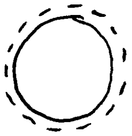
$f = [0.79 \ln(Re_D) - 1.64]^{-2}$

$10^4 < Re_D < 5 \times 10^6$

$0.08 < \mu_D/\mu_w < 40$

discussion 11/17/09

lead pellet (sphere)



air
ρ_a, ν, K
$T_\infty = 15°C$

– in free fall
$D = 3\ mm$

Table A-4: Air @ 15°C
$\rho_a = 1.22\ kg/m^3$
$K = 25.3 \times 10^{-3}\ \frac{W}{m\text{-}K}$
$\nu = 14.8 \times 10^{-8}\ m^2/s$

Table A-7: Lead (Melting Point $= T_s = 327.2°C$)
$\rho_e = 10{,}600\ kg/m^3$, $HF = 24.5\ kJ/kg$

$Pr = 0.71$
$\mu = 178.6 \times 10^{-7}\ N\text{-}s/m^2$

air @ 327°C
$\mu_s = 306 \times 10^{-7}\ N\text{-}s/m^2$

Energy Balance:

$$E_{out} = \Delta E_{st}$$

$$q_{conv.} \times t_s = -HF \times \rho_e \left(\frac{\pi D^3}{6}\right)$$

$$h(\pi D^2)(T_s - T_\infty) = -HF\,\rho_e\left(\frac{\pi D^3}{6}\right)$$

$$\text{Height} = V \times t_s = \frac{V \cdot HF \cdot \rho_e D}{6\,h\,(T_s - T_\infty)}$$

$$F_g = F_D$$

$$\rho_e\left(\frac{\pi D^3}{6}\right) g = C_D\left(\frac{\pi D^2}{4}\right)\left(\rho_a \frac{V^2}{2}\right)$$

$$V = \left(\frac{4}{3}\frac{\rho_e}{\rho_a}\frac{gD}{C_D}\right)^{\frac{1}{2}} = \left(\frac{4}{3}\ \frac{10{,}600\ kg/m^3}{1.22\ kg/m^3}\ \frac{(9.8\ m/s^2)(0.003\ m)}{C_D}\right)^{\frac{1}{2}} = \frac{18.5}{C_D^{\frac{1}{2}}}$$

Fig. 7.8: Graph of C_D vs. Re_D

$$Re_D = \frac{VD}{\nu} = \frac{V \times 0.003\ m}{14.8 \times 10^{-6}\ m^2/s} = 202.7\,V \rightarrow V \approx 29\ m/s \text{ since } Re_D = 5{,}900$$

Whitaker Correlation:

$$\overline{Nu}_D = 2 + \left(0.4\,Re_D^{\frac{1}{2}} + 0.06\,Re_D^{\frac{2}{3}}\right) Pr^{0.4}\left(\frac{\mu}{\mu_s}\right)^{\frac{1}{4}} = 40.4$$

$$h = \frac{\overline{Nu}_D \cdot K}{D} = \frac{(40.4)(25.3 \times 10^{-3}\ \frac{W}{m\text{-}K})}{0.003\ m} = 341\ \frac{W}{m^2\text{-}K}$$

$\therefore$ Height $= 35\ m$

8.36) Water flows through a thick-walled tube

$d_i = 12\,mm$ $\quad T_\infty = 85^\circ C$ $\quad \dot{m} = 33\ \frac{kg}{h}$

$L = 8\,m$ $\quad T_{m,i} = 20^\circ C$ — inlet temp. of process fluid

$R''_{cd} = 0.002\ \frac{m^2 \cdot K}{W}$ (conduction resistance of the tube wall)

a) $T_{m,o} = ?$ Assume and justify fully developed flow and thermal conditions within tube.

Table A-6: H_2O at $T_m = 337\,K$

$C_p = 4184\ \frac{J}{kg \cdot K}$, $k = 0.6574\ \frac{W}{m \cdot K}$, $\mu_{surf}(358K) = 3.316 \times 10^{-4}\ \frac{N \cdot s}{m^2}$

$\mu = 4.415 \times 10^{-4}\ \frac{N \cdot s}{m^2}$, $Pr = 2.80$

$$\frac{T_s - T_{m,out}}{T_s - T_{m,in}} = \exp\left(-\frac{\bar{U} A_s}{\dot{m} C_p}\right)$$

$$Re_D = \frac{4\dot{m}}{\pi D \mu} = \frac{4\left(\frac{33\ kg/h}{3600\ s/hr}\right)}{\pi(0.012m)\left(4.415 \times 10^{-4}\frac{N \cdot s}{m^2}\right)} = 2210$$

$Nu_D = 3.66$, $T_s = $ const.

$$h = \frac{Nu_D k}{D} = \frac{3.66 \times 0.6574\ \frac{W}{m \cdot K}}{0.012m} = 201\ \frac{W}{m^2 \cdot K}$$

$$\frac{1}{\bar{U}} = \frac{1}{h} + R''_{cd} \rightarrow \bar{U} = \left(\frac{1}{h} + R''_{cd}\right)^{-1} = \left(\frac{1}{201\ \frac{W}{m^2 \cdot K}} + 0.002\ \frac{m^2 K}{W}\right)^{-1} = 143.1\ \frac{W}{m^2 \cdot K}$$

$$\frac{85 - T_{m,o}}{85 - 20} = \exp\left(-\frac{143.1\ \frac{W}{m^2 \cdot K} \cdot \pi \cdot 0.012m \cdot 8m}{\frac{33\ kg/hr}{3600\ s/hr} \times 4187\ \frac{J}{kg \cdot K}}\right) \rightarrow T_{m,o} = 64^\circ C$$

Eq. 8.23: $\frac{x_{FD}}{D} = 0.05\, Re_D \cdot Pr \rightarrow x_{FD} = 3.20\,m$ ✓

8.42) Cooling water flows through thin-walled tubes of a steam condenser

$D = 25.4\,mm$ $T_s = 350\,K$

$u = 1\,m/s$ $T_{m,i} = 290\,K$

$L = 5\,m$

a) $T_{m,o} = ?$ Assume $\bar{T}_m = 300\,K$

Table A6: Water @ 300K

$\rho = 997\,\frac{kg}{m^3}$, $C_p = 4179\,\frac{J}{kg\text{-}K}$, $\mu = 855 \times 10^{-6}\,\frac{kg}{s\text{-}m}$, $k = 0.613\,\frac{W}{m\text{-}K}$, $Pr = 5.83$

$\frac{L}{D} = 197$

Eq 8.45b: $\frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-\frac{\bar{h} A_s}{\dot{m} C_p}\right)$

$Re_D = \frac{\rho u D}{\mu} = \frac{(997\,\frac{kg}{m^3})(1\,\frac{m}{s})(0.0254\,m)}{855 \times 10^{-6}\,\frac{kg}{s\text{-}m}} = 29{,}618$

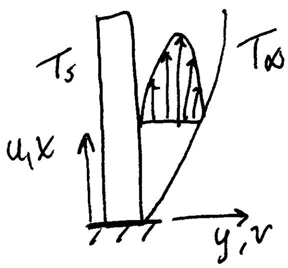
$Nu_D = 0.023\,Re_D^{4/5}\,Pr^{0.4} = 176$ $\bar{h} = \frac{Nu_D\,k}{D} = 4248\,\frac{W}{m^2 K}$

$\dot{m} = \rho u_m \left(\frac{\pi D^2}{4}\right) = 997\,\frac{kg}{m^3}\,(1\,\frac{m}{s})\left(\frac{\pi(0.0254\,m)^2}{4}\right) = 0.505\,\frac{kg}{s}$

$\frac{350 - T_{m,o}}{350 - 290} = \exp\left(-\frac{(4248\,\frac{W}{m^2 K})(\pi(0.0254\,m)(5\,m))}{0.505\,\frac{kg}{s}(4179\,\frac{J}{kg\text{-}K})}\right) \rightarrow \therefore T_{m,o} = 323\,K$

Chap. 9 11/18/09

Natural Convection



gov. eq.

1. $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} =$
2. $u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \beta g(T - T_\infty) + \nu\frac{\partial^2 u}{\partial y^2}$
3. $u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}$

$\alpha = \frac{\rho c_p}{k}$

Forced Convection	Nat. Convection	Mixed Convection
$Re = \frac{VL}{\nu}$	$Gr = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}$	$\}$
$Pr = \frac{\nu}{\alpha}$	$Pr = \frac{\nu}{\alpha}$	
$\overline{Nu} = f(Re, Pr)$	$\overline{Nu} = f(Gr, Pr)$	$\overline{Nu} = f(Re, Gr, Pr)$

Turbulence in Nat. Conv.

$\underbrace{Gr\,Pr} \approx 10^9$ $\quad \underset{\text{laminar}}{< 10^9} \quad \underset{\text{turbulent}}{\geq 10^9}$

$$\boxed{Ra = Gr\,Pr = \frac{g\beta(T_s - T_\infty)L^3}{\nu\alpha}}$$

Solving Nat. Conv. Problems

1. Identify Geometry
2. Pick the right Correlation
3. Calculate, Gr, Pr, Ra
4. Nu
5. h

} Known Temp.

1.
2.
3. Guess $\bar{h} \rightarrow q'' = h\Delta T \longrightarrow \Delta T = T_s = T_\infty$
4. Gr, Pr, Ra
5. Nu
6. Calculate $\bar{h}$
7. Check $\bar{h}$ guessed vs $\bar{h}$ calc.

} Known q''

(Ex 9.2) 1. $T_f = \frac{T_s + T_\infty}{2} = 400\ K \xrightarrow{A4} k = 33.8e^{-3}$, $\nu = 26.4e^{-6}\ m^2/s$, $\beta = \frac{1}{T_f} = 0.0025\ \frac{1}{K}$ $\quad Pr = 0.69$

$Gr = ?$

$Pr = 0.690$
$Ra = Gr\,Pr = 1.813e^{+9}$

Verticle Plate → Eq. 9.26 → $\overline{Nu} = 147$

$q = \bar{h} A_s (T_s - T_\infty) \qquad \overline{Nu} = \frac{\bar{h} L}{k} \rightarrow \bar{h} = 7.0\ \frac{W}{m^2\text{-}K}$

$\therefore\ q = 1060\ W$

Chap. 12 – 11/23/09

Radiation Heat Transfer

- H.T. by electromagnetic radiation
- All bodies above 0 K emit radiation

λ - wavelength $\qquad c = \lambda \nu$

ν - frequency $\qquad E = h\nu$, $h = 6.625 \times 10^{-34}$ J-s

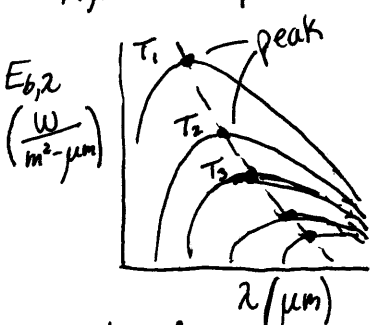
h - Plank's constant

Electromagnetic Energy Distribution:

$$E_{b\lambda} = \frac{8\pi h c \lambda^{-5}}{e^{hc/\lambda kT} - 1} = \frac{C_1 \lambda^{-5}}{e^{C_2/\lambda T} - 1}$$

K - boltzman's constant
$K = 1.38 \times 10^{-23}$ J/molecule-K

Fig. 12.12: Spectral Blackbody Emissive Power



$$\int_0^\infty E_{b,\lambda}\, d\lambda = \sigma T^4$$

$T_1 > T_2 > T_3$ $\qquad$ Peak $E_{b,\lambda}$ at lower λ as T increases

Location of peak $E_{b,\lambda}$ at $\frac{dE_{b,\lambda}}{d\lambda} = 0$

Wien's Law:

$\lambda_{max} \cdot T = 2897\ \mu m\text{-}K$

- Radiation can be absorbed, reflected, or transmitted

Sun surface $T = 5{,}800\ K \rightarrow \lambda_{max} \approx 0.5\ \mu m$
Human bodies $T = 300\ K \rightarrow \lambda_{max} \approx 10\ \mu m$